國立中央大學 110 學年度碩士班考試入學試題

所別： 經濟學系 碩士班 不分組(一般生)　　共 2 頁 第 1 頁

科目： 統計學

本科考試禁用計算器　　＊請在答案卷(卡)內作答

本試卷所有題目均為計算題(第 5 題除外)，請詳述計算過程，否則不予給分

1. (15 points) Suppose X and Y are random variables with joint density $f(x,y) = x^2 + xy/3,\ x \in [0,1], y \in [0,2]$.

   (a) (5 points) Find the joint distribution function.
   (b) (5 points) Find the conditional expectation of Y given X.
   (c) (5 points) Find the conditional variance of Y given X

2. (15 points) Consider two independent Poisson random variables X and Y with means 1.5 and 2 respectively. (hint: a random variable $X \sim \text{Poisson}(\lambda)$ if and only if $f_X(x) = \exp(-\lambda)\lambda^x/x!$)

   (a) (6 points) Find $E(Y|X+Y=4)$.
   (b) (9 points) Find $\text{Var}(X-Y|X+Y=3)$

3. (20 points) Suppose $X_i's$ are independently and identically distributed by $\text{Uniform}(0,\theta),\ i = 1,2,\dots,n$.

   (a) (5 points) Find the method of moments estimator $\hat{\theta}_{MM}$ for θ.

   (b) (5 points) Let $X_{(1)} = \min\{X_1, X_2, \dots, X_n\}$. Find the pdf for $X_{(1)}$.
   (c) (10 points) Consider the following hypotheses: $H_0: \theta = 3$ vs $H_A: \theta < 3$

   Now consider a test that rejects the null hypothesis when $X_{(n)} \le 2.5$ is observed, where $X_{(n)} = \max\{X_1, X_2, \dots, X_n\}$. Find the probability of type I error and the power function, $K(\theta)$, of this test.

4. (28 points) Consider the following linear regression model describing NCU students' weekly expenditure on coffee as a function of weekly income (NT $).

$$Y_i = \alpha + \beta X_i + u_i$$

注意:背面有試題

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Based on a surveyed dataset, we obtain the following estimation result

$$\hat{Y}_i = (\ a\ ) + 0.12\, X_i$$

se= (30) (0.012)

t= (6) ( ) n=20, $\hat{\sigma}^2 = 100$, TSS=3,000

(a) (8 points) Please derive the OLS estimators, $\hat{\alpha}$ and $\hat{\beta}$

(b) (4 points) Please calculate the estimated coefficient (a) .

(c) (8 points) Please test H_0 : β=0 and then interpret the economic meaning of $\hat{\beta}$ (0.12)

(d) (8 marks) Please calculate the R-square and interpret this number.

5. (22 points) Suppose the model of interest is $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$. The OLS estimation may suffer some problems, because assumptions of classical linear regression model (CLRM) do not hold.

(a) (10 points) If the estimation results show a high R^2 value but few significant t ratios.

i. (6 points) What problem causes this result and why?

ii. (4 points) How do you remedy this problem?

(b) (12 points) If the assumption of classical linear regression model, $E(u_i u_j) = 0, i \neq j$ does not hold.

i. (4 points) What problem the OLS estimation suffers?

ii. (4 points) Assuming $u_t = \rho u_{t-1} + \varepsilon_t$, $-1 \leq \rho \leq 1$, the Durbin-Watson d test can be used to test this problem. Please show the Durbin-Watson d statistic.

iii. (4 points) If we find d=3.5. Please comment on the test result.

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t Distribution Table

Each entry in the table shows the value (t_d) of a *t*-distributed random variable X with degrees of freedom d, given the upper tail probability being α. That is, $P(X > t_d) = \alpha$.

	Probability of Upper Tail (α)					
Degrees of Freedom (d)	0.2	0.1	0.05	0.025	0.01	0.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.92	4.303	6.965	9.925
3	0.978	1.638	2.353	3.182	4.541	5.841
4	0.941	1.533	2.132	2.776	3.747	4.604
5	0.92	1.476	2.015	2.571	3.365	4.032
6	0.906	1.44	1.943	2.447	3.143	3.707
7	0.896	1.415	1.895	2.365	2.998	3.499
8	0.889	1.397	1.86	2.306	2.896	3.355
9	0.883	1.383	1.833	2.262	2.821	3.25
10	0.879	1.372	1.812	2.228	2.764	3.169
11	0.876	1.363	1.796	2.201	2.718	3.106
12	0.873	1.356	1.782	2.179	2.681	3.055
13	0.87	1.35	1.771	2.16	2.65	3.012
14	0.868	1.345	1.761	2.145	2.624	2.977
15	0.866	1.341	1.753	2.131	2.602	2.947
16	0.865	1.337	1.746	2.12	2.583	2.921
17	0.863	1.333	1.74	2.11	2.567	2.898
18	0.862	1.33	1.734	2.101	2.552	2.878
19	0.861	1.328	1.729	2.093	2.539	2.861
20	0.86	1.325	1.725	2.086	2.528	2.845
21	0.859	1.323	1.721	2.08	2.518	2.831
22	0.858	1.321	1.717	2.074	2.508	2.819
23	0.858	1.319	1.714	2.069	2.5	2.807
24	0.857	1.318	1.711	2.064	2.492	2.797
25	0.856	1.316	1.708	2.06	2.485	2.787
26	0.856	1.315	1.706	2.056	2.479	2.779
27	0.855	1.314	1.703	2.052	2.473	2.771
28	0.855	1.313	1.701	2.048	2.467	2.763
29	0.854	1.311	1.699	2.045	2.462	2.756
30	0.854	1.31	1.697	2.042	2.457	2.75