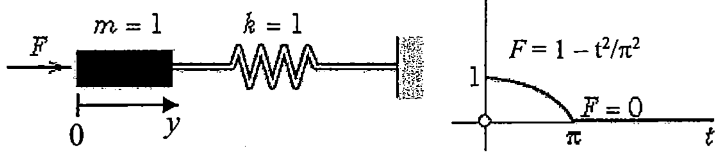
台灣聯合大學系統109學年度學士班轉學生考試試題

科目 工程數學　　類組別 A5　　共 1 頁 第 1 頁

1. Find the solutions for the following ordinary differential equations (ODEs):
   (a) $y'+xy = xy^{-1}, \quad y(0) = 3.$ (5%)
   (b) $x^2y'' - 3xy' + 3y = 3\ln x - 4, \; y(1) = 0, \; y'(0) = 1$ (5%)

2. Solve the following initial value problem using the method of Laplace Transformation:
   $y'' - y = t, y(0) = 1, y'(0) = 1$ (8%)

3. Referring to the following figure, find the equation that can describes (models) the motion of the mass as a function of time, $y(t)$, and solve the modeling equation if the initial position and velocity of the mass are both zeros, i.e., $y(0) = 0$ and $y'(0) = 0$. (10%)



4. Find the eigenvalues and eigenvectors (7%)

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

5. Solve the following problems (Each 5%) (15%)
(a) If a curve can be described using parametric representation as $\vec{\mathbf{r}}(t) = [a\cos^3 t, a\sin^3 t]$, find the arc length, s, if the parameter goes from $t = 0$ to $t = \pi/2$.

(b) Find a unit normal vector of the surface $16x^2 - y^2 = 399$ at the point $P: \left(\frac{1}{8}, 1\right)$.

(c) Find the directional derivative of $f = x^2 + y^2 + z^2$ at P: $P:(2, 2, -1)$ in the direction $\vec{\mathbf{a}} = [1,1,3]$.

6. A is a $n \times n$ matrix, $\vec{e}_p$ is a unit vector of size $n \times 1$ with its non-zero at the pth row. Let $A\vec{x} = \vec{e}_1$, $A\vec{y} = \vec{e}_n$, and $A\vec{z} = 2\vec{e}_1 + 3\vec{e}_n$. Find the solutions $\vec{z}$ in terms of $\vec{x}$ and $\vec{y}$. (10%)

7. Let S be the surface of a cube with length 1 on each side, centered at $[0,0,0]$. Let $\vec{u} = [x, y, z]$. Denote $\vec{n}$ as the unit out-normal of S. Calculate $\int_S \vec{u} \cdot \vec{n}\, dA$ using divergence theorem. (10%)

8. Solve the PDE $u(x,t)$: $u_{tt} = u_{xx}, 0 \le x \le 1, t \ge 0$ with BCs: $u(0,t) = u(1,t) = 0$, and ICs: $u(x,0) = \sin(\pi x) + 3\sin(3\pi x)$, $u_t(x,0) = 0$. (15%)

9. Solve the PDE $u(x,t)$: $u_t = u_{xx} - u, 0 \le x \le 1, t \ge 0$ with BCs: $u(0,t) = u(1,t) = 0$, and IC: $u(x,0) = \sin(\pi x)$. (15%)