國立中央大學111學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 工業管理組(一般生)　　共2頁　第1頁

科目： 作業研究

1. (計算題，15%)

Use the simplex method or the dual-simplex method to solve the following linear programming model and report an optimal solution (if there exists one). Please show detailed calculation processes; **no other solution methods** are allowed.

$$
\begin{aligned}
&\text{maximize } x_1 + 2x_2 - 3x_3 - 3x_4 \\
&\text{subject to} \\
&\quad 2x_1 - 11x_2 + 12x_3 - 10x_4 \le 50 \\
&\quad 2x_1 - 15x_2 + 4x_3 - 5x_4 \ge 10 \\
&\quad x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

2. (申論問答題，35%)

Given a number of cities with a direct link connecting any two cities (任何兩個城市之間都有直接路徑), the objective of the **traveling salesman problem** is to identify a travel tour for the salesman so he or she can visit every city once and only once with the shortest possible total distance. Suppose $\{1,2,\ldots,\mathrm{S}\}$ denotes the set of all cities; that is, there are a total S cities that the salesman must all visit. Then, this problem can be formulated as the following model with $\mathbf{d}_{ij}$ denoting the known travel distance between city i and city j (that is, we know the value of each $\mathbf{d}_{ij}$). Furthermore, x_{ij} denotes the binary decision variable such that $x_{ij} = 1$ indicates that link (i,j) is selected for the salesman to travel and $x_{ij} = 0$ indicates that link (i,j) is not selected for the salesman.

$$
\text{minimize} \sum_{i=1}^{\mathrm{S}} \sum_{\substack{j=1,\\ j\neq i}}^{\mathrm{S}} \mathbf{d}_{ij} x_{ij}
$$

subject to

$$
\sum_{\substack{j=1,\\ j\neq i}}^{\mathrm{S}} x_{ij} = 1 \text{ for any } i \in \{1,\ldots,\mathrm{S}\}
$$

$$
\sum_{\substack{i=1,\\ i\neq j}}^{\mathrm{S}} x_{ij} = 1 \text{ for any } j \in \{1,\ldots,\mathrm{S}\}
$$

$$
\sum_{i\in\mathbf{K}} \sum_{\substack{j\in\mathbf{K},\\ j\neq i}} x_{ij} \le |\mathbf{K}| - 1 \text{ for any subset } \mathbf{K} \text{ of } \{1,\ldots,\mathrm{S}\} \text{ (but } \mathbf{K} \text{ is not } \{1,\ldots,\mathrm{S}\})
$$

注意:背面有試題

(continued from the previous page) The first constraint ensures that from any city i, the salesman will travel to one and only one of the other S-1 cities. On the other hand, the second constraint ensures the opposite situation. As to the third constraint …

2.a (15%) What is this constraint ensuring? (說明第三限制式的含意，亦即，它在限制什麼？)
2.b (20%) Why is this constraint needed? (為何需要第三限制式？)

Note that in the third constraint, $|\mathbf{K}|$ denotes the number of cities contained in **K**.

3. Consider a M/M/1/k queueing model, where k is the maximum number of customers allowed in system, λ is the arrival rate, μ is the service rate, and $\frac{\lambda}{\mu} < 1$.

(a) Determine the steady-state probability $p_n, n = 0, 1, \ldots, k$. (5 points)

(b) Show that the expected number of customers in system, L, is

$$L = \frac{u}{1-u} - \frac{(k+1)u^{k+1}}{1-u^{k+1}}$$

, where $u = \lambda/\mu$. (15 points)

4. Mr. Brown is a coffee addict. He keeps switching between three brands of coffee, say, A, B, and C, from week to week according to a discrete-time Markov chain with the following transition probability matrix:

$$P = \begin{bmatrix} .1 & .2 & .7 \\ .1 & .5 & .4 \\ .3 & .2 & .5 \end{bmatrix}$$

(a) If he is using brand A this week (i.e., week 1), what is the probability distribution of the brand he will be using in week 4? (5 points)

(b) What is the limiting distribution of the brand he will use? (10 points)

5. (15 points) Let X_n be the sum of first n outcomes of tossing a four-sided die with range of values from 1 to 4, repeatedly and independently. Compute

$$\lim_{n\to\infty} P(X_n \text{is divisible by } 5)$$