

國立中央大學 111 學年度碩士班考試入學試題

所別： 光電類

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科目： 工程數學

Choose the correct answer (50 %): 單選題 每題 5 分

- (1) For an equation given as  $\frac{dy}{dx} = x$  with  $y(-1) = 1$ , what is the relationship between  $x$  and  $y$ ?  
 (A)  $y = 0.5x^2 + 1$  (B)  $x^4 - 4x - 1 = y^3 - 2y^2 + 4\log|y|$  (C)  $y = 0.5x^2 + 0.5$   
 (D)  $x^4 - 4x - 2 = y^4 - 2y^2 + 4\log|y|$  (E)  $3y = 4x^2 + 1$  (F)  $2y = x^2 + 1$  (G)  $x^4 - 4x - 1 = y^4 - 2y^3 + 4\log|y|$ .
- (2) Consider a tank with two chemical solutions separated by a porous membrane,  $f(t)$  is the time-dependent concentration of the solution on one side of the membrane and  $u(t)$  is the concentration on the other side. If  $f(t)$  and  $u(t)$  are related by the equation:  $\frac{du}{dt} = 2[f(t) - u(t)]$ ,  $u(0) = 3$ , and  $f(t) = 4$  for  $0 \leq t \leq 10$  sec;  $f(t) = 1$  for  $t > 10$  sec, what is  $u(t)$  when  $t = 5$  sec?  
 (A) 7 (B) 6 (C) 5 (D) 4 (E) 3 (F) 2 (G) 1 [f(t) and u(t) are in arbitrary unit]
- (3) If  $\frac{dy}{dx} = xy + x$  and  $y = 0$  when  $x = 0$ , what is  $y$  when  $x = 1$ ? (A) 5 (B) 0.65 (C) 0.27  
 (D) -1 (E) 0 (F) 1.33 (G) 0.5
- (4) If  $c(t) = 1 - e^{-t/100}$  and  $\frac{ds}{dt} = 3c(t) - 3\frac{s(t)}{100}$ , and  $s(0) = 10$ , what is  $s(t)$ ?  
 (A)  $150 - 150e^{-t/100} + 60e^{-3t/100}$  (B)  $100 - 150e^{-t/150} + 60e^{-3t/100}$  (C)  $100 - 250e^{-t/100} + 60e^{-3t/100}$   
 (D)  $10 - 150e^{-t/100} + 60e^{-3t/100}$  (E)  $100 - 150e^{-t/100} + 60e^{-3t/200}$  (F)  $100 - 350e^{-t/100} + 60e^{-3t/100}$   
 (G)  $100 - 150e^{-t/100} + 60e^{-3t/100}$ .
- (5) For the differential equation:  $dy/dx = x(1 + y^2)$ , if  $y = 1.558$  when  $x = 1$ , what is  $y(x)$ ?  
 (A)  $\tan(0.5x^2 + 0.5)$  (B)  $\tan(0.5x^2 + 1)$  (C)  $\tan(x^2 + 0.5)$  (D)  $2 \tan(0.5x^2 + 0.5)$   
 (E)  $\tan(0.5x^2)$  (F)  $2.5 \tan(0.5x^2 + 0.5)$  (G)  $\tan(0.5x^2 + 0.2)$
- (6) Given  $D = \frac{d}{dx}$ , what is  $(D^2 + D - 1)\sin(x)$ ? (A)  $2\cos(x) - \sin(x)$  (B)  $\cos(2x) - 2\sin(x)$   
 (C)  $3\cos(x) - \sin(x)$  (D)  $\cos(x) - 2\sin(x)$  (E)  $\cos(x) - 3\sin(x)$  (F)  $\cos(x) - 2\sin(2x)$   
 (G)  $\cos(3x) - 2\sin(x)$
- (7) If  $c_1 e^{(a+ib)x} + c_2 e^{(a-ib)x} = e^{ax} \cos(bx)$ , what are  $c_1$  and  $c_2$ ? (A)  $c_1 = 1; c_2 = 1$  (B)  $c_1 = 0.5; c_2 = 1$   
 (C)  $c_1 = 1; c_2 = 0.5$  (D)  $c_1 = 0; c_2 = 0.5$  (E)  $c_1 = 0.5; c_2 = 0$  (F)  $c_1 = 0.5; c_2 = 0.5$   
 (G)  $c_1 = 1.5; c_2 = 0.5$
- (8) What is the differential equation that has the solution of "sin(2x)"? (A)  $y'' + 4y' = 0$   
 (B)  $y'' + 4y = 0$  (C)  $y'' + 4y = 1$  (D)  $y' + 4y = 0$  (E)  $y'' + 4y' = 2$  (F)  $y'' + 5y = 0$  (G)  $5y'' + 4y = 0$   
 ( $y' = dy/dx; y'' = d^2y/dx^2$ )
- (9) If  $y = \frac{1}{3}x^2$  is a solution of  $y' + y/x = x$ ;  $y = \frac{1}{4}x^3$  is a solution of  $y' + y/x = x^2$ , what is the solution of  $y' + y/x = 3x - 5x^2$ ? (A)  $x^2$  (B)  $x^2 + x^3$  (C)  $x^3$  (D)  $x^2 + 2x^3$  (E)  $x + 2x^3$  (F)  $x^2 + 1.2x$  (G)  $x^2 - 1.25x^3$
- (10) Find an exponential multiplier  $M(x)$  for  $y' + 2y$  such that  $M(x)$  times  $(y' + 2y)$  can be written in the form  $(d/dx)[M(x)y]$ . (A)  $e^x$  (B)  $e^{3x}$  (C)  $e^{2x}$  (D)  $e^{0.2x}$  (E)  $e^{4x}$  (F)  $e^{5x}$  (G)  $e^{0.5x}$

注意: 背面有試題

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科目：工程數學

計算題 Solve the following problems (50%) (請務必寫出計算過程)

11. (10pt)  $A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & -1 \end{bmatrix}$ , Calculate the eigenvalues and eigenvectors of  $A$ .

Here  $i = \sqrt{-1}$  is the unit of imaginary number.

12. Calculate the divergence and curl of the following vector fields:

(a) (5pt)  $\mathbf{V} = \hat{k} \times \nabla r^2$ , here  $r^2 = x^2 + y^2 + z^2$  and  $\hat{k}$  is the unit vector along the z-axis.

(b) (5pt)  $\mathbf{W} = x(y-z)\hat{i} + y(z-x)\hat{j} + z(x-y)\hat{k}$ .

13. A periodic function is defined by the sum of infinite terms:

$$f(x) = \sum_{n=-\infty}^{\infty} e^{-|x-2n\pi|}.$$

(a) (2pt) What is the period? Is this function an even function or an odd function?

(b) (8pt) Find the Fourier series expansion of this function.

14. (10pt) The Gaussian integral formula is

$$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right),$$

where  $a > 0$  is a positive real number, and  $\exp(\alpha) = e^\alpha$  is the exponential function. Use the Gaussian integral formula to find the Fourier transform of the function

$$f(x) = e^{-x^2} \cos x.$$

15. (10pt) Solve the following initial value problem. Here the  $\delta(t-1)$  and  $u(t-2)$  are the Dirac delta function and Heaviside step function, respectively.

$$y'' - 2y' + y = \delta(t-1)u(t-2), \quad y(0) = 1, \quad y'(0) = 0.$$