國立中央大學111學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)
統計研究所 碩士班 不分組(在職生)

科目： 數理統計

計算題應詳列計算過程，無計算過程者不予計分

1. Let Y be a positive random variable such that $\log Y$ is of the normal distribution with mean μ and variance σ^2.
   (a) (5 %) Find the distribution of $1/Y$.
   (b) (10 %) Find the mean and variance of $1/Y$.
2. (10%) Let $X_1, X_2, \ldots, X_n$ be a random sample taken from the exponential distribution with mean $\lambda > 0$. Find the asymptotic distribution of the maximum likelihood estimate (MLE) of $\theta = \Pr(X_1 > 1)$ as $n \to \infty$.
3. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from the normal distribution with mean $\mu - \nu$, where $\mu > \nu$ and variance 1.
   (a) (10%) Can you find the maximum likelihood estimates (MLE) of μ and ν? Give your reason.
   (b) (10%) Find the best unbiased estimator of $\mu^2 - \nu^2$ with minimum variance, if it is known that $\mu = -3\nu$.
4. (10 %) Let $Y_1, Y_2, \ldots, Y_5$ be a random sample with probability density function

$$f(y) = \frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1}, 0 \le y \le \theta,$$

   and zero otherwise, where $\beta > 0$ is known and $\theta > 0$ is unknown. Find an equal-tailed $(1-\alpha)100\%$ confidence interval for θ.
5. (10 %) The reaction of an individual to stimulus in a psychological experiment may take one or two forms, A or B. If an experimenter wishes to estimate the probability that a person will react in manner A, how many people must be included in the experiment? Assume that the experimenter will be satisfied if the error of estimation is less than 0.04 with probability equal to 0.95. (Note that $\frac{1}{\sqrt{2\pi}}\int_{-1.645}^{\infty} e^{-t^2/2}\mathrm{dt} = 0.95$ and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-1.96} e^{-t^2/2}\mathrm{dt} = 0.025$.)

注意:背面有試題

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6. Let Y_1 and Y_2 be independent and identically distributed random variables with probability density function $f(y) = 1$ for $\theta < y < \theta + 1$, and zero otherwise. For testing $H_0 : \theta = 0$ versus $H_a : \theta = -0.5$, consider

   Test 1: Reject H_0 if $Y_1 < .02$;

   Test 2: Reject H_0 if $Y_1 + Y_2 < c$.

   (a) (10 %) Find the value of c so that Test 2 has the same size as Test 1.

   (b) (5 %) Find the Type II error probability of Test 2 obtained in (a).

7. (10 %) Let X_1 and X_2 be independent and identically distributed random variables with probability mass function $f(x|\theta)$, where θ could be either 1, 2 , or 3, given by

x	1	2	3
$f(x\|\theta = 1)$	0.5	0.2	0.3
$f(x\|\theta = 2)$	0.3	0.4	0.3
$f(x\|\theta = 3)$	0.4	0.1	0.5

   Find the MLE of θ based on two observations X_1 and X_2.

8. (10 %) Suppose that $Y_1, Y_2, \ldots, Y_5$ are a random sample from the Poisson distribution with mean λ. Derive an exact 95% confidence interval for λ if $(y_1, \ldots, y_5) = (1, 0, 1, 2, 1)$.

Table of $\chi^2_{\nu;\alpha}$ for the Chi-square distribution of ν degrees of freedom: $\Pr(\chi^2_\nu > \chi^2_{\nu;\alpha}) = \alpha$.

		ν					
		4	5	6	10	11	12
	.025	11.143	12.833	14.449	20.483	21.920	23.337
	.05	9.487	11.071	12.592	18.307	19.675	21.026
α	.90	1.064	1.610	2.204	4.865	5.578	6.304
	.95	0.711	1.145	1.635	3.940	4.575	5.226
	.975	0.484	0.831	1.237	3.247	3.816	4.404

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