國立中央大學 111 學年度碩士班考試入學試題

所別： 資工類　　共 6 頁　第 1 頁

科目： 資料結構與演算法

一、複選題(50 分)，每題 5 分，全對才給分，答錯倒扣 1 分，倒扣至複選題 0 分為止

1. Which of the following statements are true?
   (A) Consider the convert the infix expression a*(b+c/d)*(e-f)+g to its postfix form. There are at most 4 tokens in this stack at any moment during the conversion.
   (B) The postfix form of infix expression (a+b-c)*(d-e) is abc+-de-*
   (C) The infix expression of postfix expression 3k-2m4+*+pab-*/ is ((3-k)+2(m+4))/(p*(a-b))
   (D) Consider the convert the infix expression a*(b+c/d)*(e-f) to its postfix form. There are at most 3 tokens in this stack at any moment during the conversion.

2. Which of the following statements are true?
   (A) Dynamic programming is a technique that avoids the recursive explosion.
   (B) Overlapping recursive calls tend to yield exponential algorithm.
   (C) Greedy algorithm makes locally optimal decision at each step.
   (D) Divide and conquer is a type of recursive algorithm. The recursion is the divide part.

3. Which of the following statements about n-element AVL trees are false?
   (A) The time complexity of rebalancing rotation after deleting an element is θ(1og n).
   (B) The time complexity of rebalancing rotation after insertion is θ(1og n).
   (C) If the AVL tree has height = h, then n <= F_{h+2} -1, where F_{h+2} is the Fibonacci number, i.e., $F_{h+2}=F_{h+1}+F_h$.
   (D) h_L - h_R should be smaller than 1, where h_L and h_R denote the height of the left subtree and the right subtree, respectively.

4. Consider a height-biased leftist tree (HBLT). Let **w**(x) be the number of internal nodes in the subtree with root x. Which of the following statements are false?
   (A) The length of the lefttmost path from internal node x to an external node must be no greater than $\log_2(\mathbf{w}(x)+1)$.
   (B) The height of the subtree with root x must be no greater than $\log_2(\mathbf{w}(x)+1)$.
   (C) Removing minimum of an n-element min HBLT is done in time O(log n).
   (D) Melding two n-element min HBLTs is done in time θ(log n).

5. Which of the following statements are false?
   (A) The function **F1** below is executable (i.e., terminable) for all positive integer x.

```
Int F1(int x)
{
      If x is even then
          return x/2;
      else
          return F1(F1(3x+2));
}
```

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(B) When input data are 21 and 12, the output of function **F2** below is 4.

```
Int F2(int x, int y)
{
    If y=0 then
      return x;
    else
      return F2(y, x mod y);
}
```

(C) Consider the function **F3** below. **F3**(5) is 5.

```
Int F3(int x)
{
    Int p, q;

    If x <= 2 then
      return x;
    else
      p=F3(x-2);
      q=F3(x-3);
      return p+q;
}
```

(D) None of above.

6. We have a 2-dimension integer array arr[5][5],
   Which of the followings are equal to arr[2][3]?
   (A) *(*arr+13) (B) (*(arr+2))[3] (C) *((*(arr+2))+3) (D) (*arr+2)[3]

7. Given two devices, A and B, connected through a black box. When device A transmits 5 messages, m1, m2, m3, m4, and m5, to device B sequentially, which of the following statements are false?
   (A) The black box must be a queue when device B receives m1, m2, m3, m4, and m5 sequentially.
   (B) The black box could be a stack when device B receives m3, m2, m1, m4, and m5 sequentially.
   (C) The black box could be a max heap when device B receives m3, m2, m5, m4, and m1 sequentially.
   (D) The black box neither a stack nor queue when device B receives m3, m2, m1, m4, and m5 sequentially

8. Given an empty hash Table T of size 17 and a set {23, 12, 34, 46, 28, 11, 6, 7, 0, 33, 13, 45} to hash into the table. Suppose that the hash function **h**(k , i) = (**h**(k) + i) mod 17, where i = 0 to 16. After deleting 46 from the hash table, which keys should modify their positions?
   (A) 11 (B) 0 (C) 33 (D) 45 (E) 13

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9. The pseudo code below aims to reverse a linked list.

```
struct Node {
    int data;
    struct Node* next;
};
typedef struct Node* NPtr;

int main()
{
    NPtr head = NULL; /* Start with an empty linked list */
    push(&head, …); /* insert items into the linked list */
    reverse(&head); /* reverse the linked list */
 }

void reverse(NPtr* Href)
{
    NPtr P = NULL;
    NPtr C = *Href;
    NPtr N = NULL;

    while (    (1)    ) {
        N = C->next;
        C->next =    (2)    ;
        P = C;
        C = N;
    }
        (3)
}
```

Which of the following statements are true?

(A) Blank (1) should be C* != NULL.

(B) Blank (2) should be N.

(C) Blank (3) should be *Href = C;

(D) Statement P = C; moves the pointer to the next.

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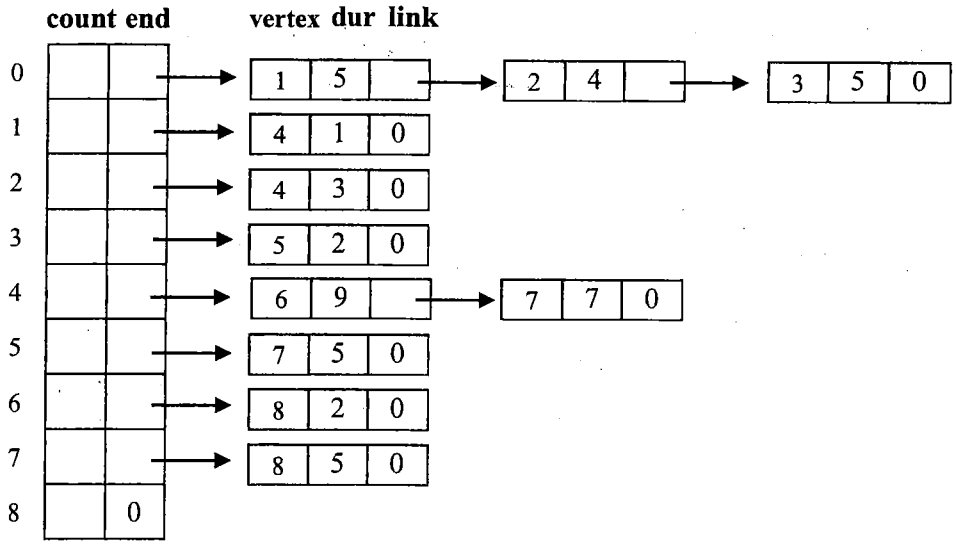
10. The adjacency list below is for an AOE network. The **end** field points to a list of adjacent vertices, **dur** field is the duration of the activity, **link** field points to another adjacent vertex, **vertex** field is the id of the adjacent vertex, **count** field is the number of immediate predecessors. Which of the following statements are true?
    (A) The total duration of the critical path is 17.
    (B) Path 0, 1, 4, 6, 8 is not a critical path.
    (C) Count fields from vertex 0 to vertex 8 should be 0, 1, 1, 1, 2, 2, 1, 2, 2.
    (D) The latest time that event 4 (i.e., vertex 4) can occur is 7.



二、多選題(10 分)，每題 5 分，每個選項答對得 1 分，答錯倒扣 1 分，倒扣至多選題 0 分為止

11. Choose the correct statement(s):
    (A) The insertion sort algorithm has the worst-case time complexity $O(n^2)$, the average-case time complexity $O(n^2)$, and the best-case time complexity $O(n)$.
    (B) The merge sort algorithm has the worst-case time complexity $O(n \log n)$, the average-case time complexity $O(n \log n)$, the best-case time complexity $O(n \log n)$, and the space complexity $O(n)$.
    (C) The quick sort algorithm has the worst-case time complexity $O(n \log n)$, the average-case time complexity $O(n \log n)$, and the best-case time complexity $O(n \log n)$.
    (D) The heap sort algorithm has the worst-case time complexity $O(n \log n)$, the average-case time complexity $O(n \log n)$, the best-case time complexity $O(n \log n)$, and the space complexity $O(n)$.
    (E) The bubble sort algorithm has the worst-case time complexity $O(n^2)$, the average-case time complexity $O(n^2)$, and the space complexity $O(1)$.

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12. Choose the correct statement(s):
(A) It has been proved that there exists no deterministic algorithm that can solve any given NPC problem with a polynomial time complexity in the worst case.
(B) If we can prove that the worst-case problem lower bound of an NPC problem is of a polynomial order, then we can prove that NP≠P.
(C) If we can prove that an existing NP-hard problem is polynomially reducible to a problem X, then X is NP-hard.
(D) If a problem is proved to be an NP-hard and NP problem, then it is an NPC problem.
(E) If a problem X can be solved by a non-deterministic algorithm taking a polynomial time complexity in the worst case, then X is an NP problem.

三、問答題(40 分) 請用深色筆書寫，勿用鉛筆

1. Given a weighted connected undirected graph $G=(V, E)$ with node set V and edge set E, a subgraph M of G is called a spanning tree of G if $M=(V, T)$, $T \subseteq E$, and $|T|=|V|-1$. M is called a minimal spanning tree (MST) of G if M is a spanning tree of G with the minimal total weight. Below is the Prim's MST algorithm to output the MST of a given graph G.

Algorithm: Prim's MST algorithm
Input: A weighted connected undirected graph $G=(V, E)$, where $|V|=n$
Output: The MST $M=(V, T)$ with the minimal total weight, where $|T|=n-1$
1: $T \leftarrow \phi$
2: $X \leftarrow \{w\}$, where w is an arbitrary node in V
3: **while** $|T| < n-1$ **do**
4: select an edge $(u, v) \in E$ with the minimal weight such that $u \in X$ and $v \in (V-X)$
5: $T \leftarrow T \cup \{(u, v)\}$
6: $X \leftarrow X \cup \{v\}$
7: **return** $M=(V, T)$

Please follow the form of the Prim's MST algorithm to
(1) write the well-known Kruskal's MST algorithm (9%) and
(2) analyze the worst-case time complexity of the Kruskal's MST algorithm (6%).
Note that you should strictly follow the form of the Prim's MST algorithm to write the Kruskal's MST algorithm; otherwise, you will lose some points.

2. (13%) You are going on a long hiking trip. You start on the road at kilometer post 0. Along the way there are n hotels, at kilometer posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination. You'd ideally like to travel 20 kilometers a day, but this may not be possible. If you travel x kilometers during a day, the penalty for that day is $(20-x)^2$. You want to plan your trip so as to minimize the total penalty. Give an algorithm that determines the optimal sequence of hotels at which to stop. (Note: This problem can be solved by a dynamic programming algorithm. If you use a dynamic programming algorithm to solve this problem, please write down the recursive formula of the algorithm, describe the meaning of the notations you used in the formula, give the initial value settings, and analyze the time complexity of the algorithm.)

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3. (12%) For a set of variables $x_1, x_2, \ldots, x_n$, you are given some equality constraints, of the form "$x_i = x_j$" and some disequality constraints, of the form "$x_i \neq x_j$". Is it possible to satisfy all of them? For instance, the constraints :

   $x_1 = x_2$,; $x_2 = x_3$; $x_3 = x_4$; $x_1 \neq x_4$;

   cannot be satisfied. Give an efficient algorithm that takes as input m constraints over n variables and decides whether the constraints can be satisfied. Describe the data structure used by your algorithm, and analysis the time complexity of your algorithm.

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