所有試題均為計算題，應詳列計算過程，無計算過程不予計分。

1. (10%) Show that $y_1 = 3e^{2x} - 1$ and $y_2 = e^{-x} + 2$ are two solutions of $yy'' + 2y' - (y')^2 = 0$. Determine whether they are linearly independent or not by Wronskian test. Is it true that the general solution can be written as $y = c_1 y_1 + c_2 y_2$? Why?

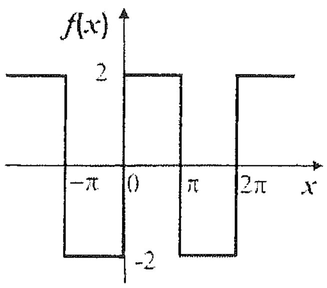
2. (10%) Find the eigen values, or the equation defining the eigen values, and the corresponding eigen functions of the following equation and boundary conditions.

$$y'' + \lambda y = 0$$
$$y(0) - 4y'(0) = 0,\ y'(1) = 0$$

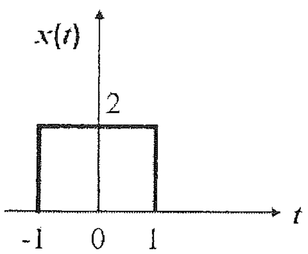
3. (15%)

(a) (8%) For the periodic function f(x) as shown, you are required use the Fourier analysis to determine the sum of the infinite series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots.$$




(b) (7%) Find the Fourier transform of the functions x(t) as shown below.

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4. (15%) For the matrix

$$M = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix},$$

(a) (10%) Find the Eigenvalues and Eigenvectors for the matrix, $M$.
(b) (5%) Show that there exists a matrix $J$, which is similar to the matrix $M$ in the Jordon form, i.e., $J = Q^{-1} M Q$.

5. (12%) Perform the singular value decomposition to the matrix

$$A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix},$$

6. (8%) Let $V$ be an inner product space and let $\hat{T}$ be a normal operator on $V$. Prove that If $\lambda_1$ and $\lambda_2$ are distinct eigenvalues of $\hat{T}$ with corresponding eigenvectors $\vec{x}_1$ and $\vec{x}_2$, then $\vec{x}_1$ and $\vec{x}_2$ are orthogonal.

7. (10%) Let $u = x^4 + y^4 - 6x^2y^2 + 2x^2 - 2y^2 + 1$. Prove that u is harmonic and find its harmonic conjugate v.

8. (10%) If C is the counterclockwise unit circle in the complex plane. Evaluate the following integral.

$$\oint_C \frac{ze^{2z}}{\left(z - \frac{i}{2}\right)^2} dz$$

9. (10%) Evaluate the following integral, where C is the ellipse $9x^2+y^2=9$ (counterclockwise).

$$\oint_C \left(\frac{ze^{\pi z}}{z^4 - 16} + ze^{\pi/z}\right) dz$$

注意:背面有試題