國立中央大學112學年度碩士班考試入學試題

所別：　化學工程與材料工程學系碩士班　　　　共5頁　第1頁
科目：　輸送現象與單元操作

計算題應詳列計算過程，無計算過程者不予計分

1. (15%) The McCabe-Thiele method is a chemical engineering technique for the analysis of binary distillation.
   (a) (7%) What is/are the major assumption(s) made in McCabe-Thiele method? At what conditions, can we make such assumption(s)?
   (b) (8%) Please draw a typical McCabe-Thiele diagram for the distillation of a binary feed and identify the equilibrium line, two operating lines, q-line, feed tray, and the compositions of feed, bottoms and distillate in the diagram.
2. (20%) A droplet of liquid A, of radius r_1, is suspended in a stream of gas B. We postulate that there is a spherical stagnant gas film of radius r_2 surrounding the droplet. The concentration of A in the gas stream is x_{A1} at $r = r_1$ and x_{A2} at the outer edge of the film, $r = r_2$. Please find the molar flux N_{Ar1} at $r = r_1$.
3. (10%) Answer the following problems
   (a) (4%, 2% for each problem) Regarding the Navier-Stokes equation:

   $$\rho\frac{Dv}{Dt}=\rho\left(\frac{\partial v}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla P+\mu\nabla^2\underline{v}+\rho\underline{g}$$

   (1) Please list the assumptions made by the Navier-Stokes equation.
   (2) Please write down the simplified equation when Reynold's number is very small. (Assume it is steady-state).
   (b) (3%, 1% for each problem) Regarding the Hagen-Poiseuille equation, which following statements are true?
   (1) Applicable to flows that are driven only by gravity.
   (2) Applicable to both Newtonian and non-Newtonian fluids.
   (3) Applicable to flows with $Re < 0.1$.
   (c) (3%, 1% for each problem) For a fluid that is Newtonian but compressible, and its flow is un-steady-state with a Reynold's number of about 10. Answer whether the following equation (Yes or No) is applicable to the fluid and the flow. No explanation is required.
   (1) Equation of continuity
   (2) Equation of motion in terms of viscous force
   (3) Stokes (creeping) flow equation
4. (20%) An incompressible (density ρ) and non-Newtonian liquid flows in an inclined circular pipe with a radius of R and length of L. The pipe is tilted with an angle of 30 degree. A schematic illustration of the system can be seen below (with the definition of system coordinates). The flow is laminar, steady-state, and it is fully developed with no edge and entrance/exit effects. P_o and P_L represent the pressures on the two ends of the tube. In this system, we have: $v_z = v_z(r), v_r = v_r = 0,$ and $v_\theta = 0$; $\tau_{\theta z} = \tau_{zz} = 0$

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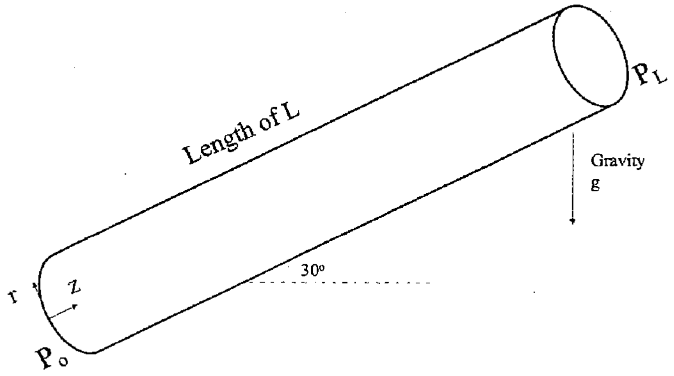
Please use the coordinates defined in the figure for derivation.



For a Bingham fluid, the shear stress of the fluid can be described as the following

$$\frac{dV_z}{dr} = 0 \quad if\ |\tau_{rz}| < \tau_o, \qquad \text{where } \tau_o = \frac{\rho g R}{8}$$

$$\tau_{rz} = \pm\tau_o - \mu\frac{dV_z}{dr} = 0;\ \text{ if } |\tau_{rz}| \geq \tau_o, \ \begin{cases} \tau_{rz} = \tau_o - \mu\dfrac{dV_z}{dr} & if\ \tau_{rz} > 0 \\ \tau_{rz} = -\tau_o - \mu\dfrac{dV_z}{dr} & if\ \tau_{rz} < 0 \end{cases}$$

(a) (10%) Should we expect fluid (Bingham fluid) to flow down under the condition of $P_o=P_L$? (No pressure drop along z-direction.) Please provide detailed to support your answer.

(b) (10%) Follow the previous question. If a pressure drop along the positive z direction ($P_o>P_L$) is provided, with a sufficiently large pressure drop we should be able to stop the fluid to flow downward and push the fluid upward. Please determine the **critical pressure drop** that allows the fluid to flow along the positive z direction (Please present the critical pressure drop P_o-P_L in terms of $\rho, g, R, L, and/or\ \mu$).

Newton's law of viscosity

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla\cdot\mathbf{v})$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]$$

in which

$$(\nabla\cdot\mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

The equation of continuity

$$[\partial\rho/\partial t + (\nabla\cdot\rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

The equation of motion in terms of τ

$[\rho D\mathbf{v}/Dt = -\nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g}]$

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

5. (35%) To satisfy the consumer market's demand for wearable devices which are simultaneously powerful, multifunctional and lightweight, electronics manufacturers have been gearing up in their R&D efforts to miniaturize their products while maintaining or even beefing up the computing powers of their devices. One key to winning this miniaturization arms race is on the ability to shrink the sizes of the chips by packaging even more transistors into ever smaller chip dies. In addition to the technical challenges associated with the electronics designs and materials development, packing unprecedented numbers of transistors into increasingly small spaces is also faced with a daunting problem: dissipation of heat generated from the transistors packed in a tiny space, a make-or-break issue for any sophisticated electronics design and industrial project aiming to miniaturize an electronic device. As an engineer, you are tasked with resolving the heat-dissipation issue for the microchip of a new sporting wearable device currently in the R&D phase of your company. You plan to make a heat sink which can dissipate heat generated from the microchip into flowing water when users of the device are swimming (or flowing air when the users are walking or running, situations that are not considered here). As the first step, you decide to construct a model system with a geometry identical to that of the real system but with a larger dimension which allows you to test with ease your heat sink designs even before the microchip comes into being. Undoubtedly, the model system, consisting of the heat sink mock-up and a flowing fluid, must reflect the heat transfer condition of the real system. And you recognize that this demands a thorough consideration from the perspective of dimensionless parameters.

(1) (2%) To ensure the similarity in heat transfer between the model and real systems, what dimensionless parameters (groups) have to be identical across the two systems?

(2) (2%) Explain the physical meanings of the dimensionless parameters in (1)?

(3) (2%) Explain the physical reasons behind the fact that the dimensionless parameters in (1) must be identical across the two systems if their heat transfer conditions are to be similar.

When the dimensionless parameters in (1) are identical across the two systems, another dimensionless parameter may give the same functional form for the two systems.

(4) (2%) Identify this dimensionless parameter and describe the information from this dimensionless parameter, which is relevant to heat transfer?

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(5) (4%) Based on the definition of the dimensionless temperature $T^* \equiv \frac{T-T_S}{T_\infty - T_S}$, where T_∞ is the free-streaming fluid and T_S is the surface temperature of the heat sink, as well as the boundary layer equations, ***conceptually*** prove that the dimensionless parameter in (4) indeed has the same functional form for the model and real systems when the dimensionless parameters in (1) are identical across the two systems.

You are informed that the dimension of the chip die is set to 1 mm across (so will the heat sink). And it is known that the water flow (at 17°C) experienced by a swimmer is of the velocity 0.8 m/s on average. You employ air at 27°C, instead of water at 17°C, as the flowing fluid in your model system.

(6) (3%) With the model system in the dimension of 10 cm, in what velocity should you operate the water flow to make the heat transfer conditions similar between the model and real systems?

(7) (2%) Is air at 27°C a right choice for your model system? Why?

Under the assumption that the heat-transfer similarity can be achieved with your choice of the conditions for the model system, you supply a fixed heat flux of 200 W/m^2 to the heat sink mock-up, which is transferred to air, and measure the surface temperature of the mock-up. In the steady-state condition, the surface temperature is measured to be 55°C.

(8) (5%) What is the convection heat transfer coefficient for the real system in operation?

(9) (3%) How will the convection heat transfer coefficient evolve along the direction parallel to the surface of the heat sink?

You decide to use sapphire as the material for the heat sink in the real system, for the sake of the aesthetic appeal to consumers. A single (one-time) operation of the microchip brings the surface temperature of the heat sink to 65°C, which is surely uncomfortable to the users.

(10) (5%) Given that both of the two faces of the heat sink are in contact with flowing water, how long does it take for the surface temperature of the heat sink to drop to a more comfortable temperature of 40°C?

(11) (5%) How much heat is transferred during the period?

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683

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Composition	Melting Point (K)	Properties at 300 K ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Properties at Various Temperatures (K) k (W/m · K)/c_p (J/kg · K) 100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum oxide, sapphire	2323	3970	765	46	15.1	450 —	82 —	32.4 940	18.9 1110	13.0 1180	10.5 1225				

TABLE A.6 Thermophysical Properties of Saturated Water^a

Temperature, T (K)	Pressure, p (bars)^b	Specific Volume (m^3/kg) $v \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	$c_{p,g}$	Viscosity (N · s/m^2) $\mu \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr	Pr_g	Surface Tension, $\sigma \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta \cdot 10^6$ (K^{-1})
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4