國立中央大學 112 學年度碩士班考試入學試題

所別： 統計研究所碩士班

科目： 數理統計 計算題應詳列計算過程，無計算過程者不予計分

1. Let X_1 and X_2 be independent random variables from exponential distribution with mean $1/\lambda$. Let U= $X_1 - X_2$ and V= X_2.
   (a) Find the joint probability density function (pdf) of U and V. (5%)
   (b) Find the pdf of U. (5%)
   (c) What is the conditional expected value of V given U=u. (10%)

2. Let X_i be the number of return visits for patient i until he/she is recovered from a disease, i =1, 2, …, n. Let θ be the probability of recovery.
   (a) Find the probability mass function (pmf) of the X_i. (5%)
   (b) Find the maximum likelihood estimate (mle) of θ, denoted by $\hat{\theta}$, based on $X_1, \ldots, X_n$. (5%)
   (c) Is $\hat{\theta}$ unbiased for θ? (5%)
   (d) Suppose that Y_i is the right-censored X_i for i=1, 2, …, n, where
   $$Y_i = X_i,\ X_i = 1, \ldots, r$$
   $$= r + 1,\ X_i = r+1, r+2, \ldots$$
   Find the pmf of the Y_i. (6%)
   (e) Find the mle of θ, denoted by $\hat{\theta}_c$, based on $Y_1, \ldots, Y_n$. (6%)

3. Elements in a population are consecutively labeled from 1 to an integer θ, the size of the population. Let $X_1, \ldots, X_n$ be a random sample from the population with pmf
   $$g(x; \theta) = 1/\theta,\ x=1, 2, \ldots, \theta$$
   $$= 0, \text{ otherwise.}$$
   (a) Find the *mle* of θ based on $X_1, \ldots, X_n$, denoted by $\hat{\theta}$. (5%)
   (b) Find a test for $H_0: \theta = \theta_0$ versus $H_1: \theta = \theta_1$ $(\theta_1 > \theta_0)$ based on $\hat{\theta}$. Specify the rejection region at significance level α. (10%)
   (c) Find the sample size required for the test in (b) so that the power reaches 1-β when $\theta_1 = \theta_0 + \Delta$. (10%)
   (d) Find the sample size for a level 0.05 test in (b) to reach 1-β = 0.90 when Δ=10. (8%)

4. Let $X_1, \ldots, X_n$ be independent Bernoulli random variables with
   $$p = P(X_1=1) = 1 - P(X_1=0).$$
   (a) Find the mle of p^2 based on $X_1, \ldots, X_n$. (10%)
   (b) Construct a level (1-α) confidence interval for p^2. (10%)