＊本科考試可使用計算器，廠牌、功能不拘

1. **(6 points)** Answer "**True**" or "**False**" for each of the following statements; explanation is not needed.

   (1) For equation of continuity

   $$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{v} = 0$$

   (a) Equation of continuity cannot be used in an unsteady-state system.

   (b) The above equation is derived from momentum balance.

   (c) The density of the fluid is invariant.

   (2) Regarding the equation of motion in terms of τ

   (a) Applicable to both incompressible and compressible fluids.

   (b) Cannot be applied to non-Newtonian fluids.

   (c) Cannot be applied to an unsteady-state problem.

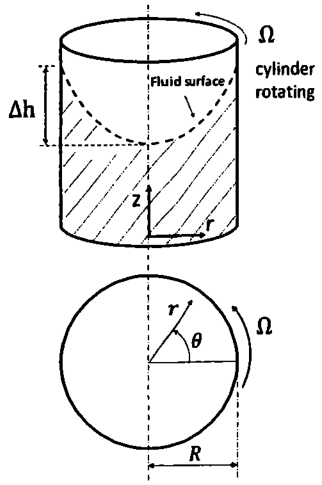
2. **(7 points)** A Newtonian and incompressible fluid is in a cylindrical container of radius R and at rest initially. There is a sudden motion applied to rotate the container at an angular speed of Ω. The time-dependent velocity profile along θ direction in this system can be determined by the following partial differential equation:

   $$\rho \frac{\partial v_\theta}{\partial t} = \mu \frac{\partial}{\partial \mathrm{r}}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right]$$

   (1) Use R and $R\Omega$ as the characteristic length and velocity, respectively. Please express the above PDE in terms of the three dimensionless variables $\tilde{r}, \tilde{v}_\theta$, and $\tilde{t}$ for variables of r, v_θ, and t, respectively. The answer should include Reynolds number, Re, which also needs to be clearly defined. **(5 points)**

   (2) If you place the same fluid in two containers with different sizes: R and 2R in radius. To maintain the dynamic similarity between the two systems, if the smaller one rotates at an angular velocity of 2Ω, what should the angular velocity of the larger container be? **(2 points – no partial credits)**

3. **(17 points)** A long cylindrical container with radius of R is rotating at an angular velocity of Ω about its own axis, depicted in the figure on the right. The cylinder axis is vertical, and it contains an incompressible and Newtonian fluid (with density ρ and viscosity μ). The rotation of the cylinder induced a height difference (Δh) of the fluid surface between the outer surface ($r = R$) and the center ($r = 0$). Assume the induced flow is steady-state and laminar, the pressure of the fluid is a function of r and z; $v_r = v_z = 0$; $v_\theta = f(r)$; p is a function of r and z; and the pressure at the surface is P_{atm}.



注意：背面有試題

國立中央大學113學年度碩士班考試入學試題

所別： 化學工程與材料工程學系 碩士班 甲組(一般生)

科目： 輸送現象與單元操作

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Answer following questions using the coordinates defined in the figure otherwise no points will be given.

(1) **(5 points)** Determine the velocity profile of the fluid in the cylinder using the given parameters.

(2) **(10 points)** Express the height difference (Δh) using the above given parameters.

(3) **(2 points)** Show the relation between the height difference of the fluid surface Δh and angular velocity Ω by plotting Δh (y-axis) vs Ω (x-axis).

4. **(15 points)** To reduce the threat of predators, the sand grouse, a bird of Kenya, will lay its eggs in locations well removed from sources of groundwater. To bring water to its chicks, the grouse will then fly to the nearest source and, by submerging the lower part of its body, will entrain water within its plumage. The grouse will then return to its nest, and the chicks will imbibe water from the plumage. Of course, if the time of flight is too long, evaporative losses could cause a significant reduction in the water content of the plumage, and the chicks could succumb to dehydration. To gain a better understanding of convective transfer during flight, wind tunnel studies were performed using molded models of the grouse. By heating the portion of the model that corresponds to the water-encapsulating plumage, an average convection heat transfer coefficient was determined. Results for different air speeds and model sizes were then used to develop an empirical correlation of the form, $\overline{Nu}_L = 0.034Re_L^{4/5}\mathrm{Pr}^{1/3}$. The effective surface area of the water-encapsulating portion of the plumage is designated as A_s, and the characteristic length is defined as $L = (A_s)^{1/2}$.

   Consider conditions for which a grouse has entrained 0.05 kg of water within plumage of $A_s = 0.04\ \mathrm{m}^2$ and is returning to its nest at a constant speed of $V = 30$ m/s. The ambient air is stagnant and at a temperature and relative humidity of $T_\infty = 37°\mathrm{C}$ and $\phi_\infty = 25\%$, respectively. If, throughout the flight, the surface A_s is covered with a liquid water film at $T_s = 32°\mathrm{C}$, what is the maximum allowable distance of the nest from the water source, if the bird must return with at least 50% of its initial water supply? (Explicitly describe your assumption(s), if any, for solving the problem; the binary diffusion coefficient for water vapor in the air D_{AB} is $0.26\times10^{-4}\ \mathrm{m^2/s}$)

5. **(15 points)** A laboratory apparatus to measure the diffusion coefficient of vapor–gas mixtures consists of a vertical, small-diameter column containing the liquid phase that evaporates into the gas flowing over the mouth of the column. The gas flow rate is sufficient to maintain a negligible vapor concentration at the exit plane. The column is 150 mm high, and the pressure and temperature in the chamber are maintained at 0.25 atm and 320 K, respectively. Given the evaporation rate of water into air being 0.257 kg/h·m^2 under the foregoing conditions, determine the value of the binary diffusion coefficient D_{AB} for the vapor–air mixture. (Explicitly describe your assumption(s), if any, for solving the problem; the vapor-air mixture may be treated as an ideal gas.)

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6. (**40 points**) Due to the worsening environmental conditions attributable to plastic wastes, governments of advanced economies have been tightening their regulations on the manufacturing and distribution of plastic products, particularly those for single uses. Under this circumstance, European Union, which boasts of the most stringent environmental regulations on the planet, revised their Waste Framework Directive and Packing Waster Directive in 2018 and, according to the new rules, will gradually require single-use plastic products to be made of biodegradable plastics. One of the industries bearing the brunt of the new regulations is fast food restaurants and the food service providers specializing in food delivery. These industries have long been heavy users of disposable plastic products; an outstanding example being the disposable containers for hot drinks. Even though fast-food giants, such as MacDonald's, have shifted to the paper-based containers (which are not eco-friendly as well), many players in the industries still use the plastic-based containers for their hot drinks, under the consideration of cost and thermal performance. As a product engineer of a biodegradable plastics manufacturer, you are tasked with the endeavor to develop a disposable coffee cup made of the biodegradable plastic, polylactic acid (PLA), with the aim of substituting it for its non-degradable plastic- and paper-based counterparts currently used in the food industry. A key step for the task is to determine the wall thickness for your new PLA coffee cup. According to a consumer survey by a trade association of the food industry, most people finish their coffee in 10 minutes and prefer the temperature of their coffee to be maintained within 10°C from its initial temperature (~90°C) in this period. You decide to begin with determining the temperature distribution within coffee immediately after it is poured into a cup.

(a) (**5 points**) Given the cylindrical geometry being the most common design for a coffee cup, the temperature distribution within coffee inside the cup should also be described in the cylindrical coordinate. In this context, the spatial and temporal evolution of temperature for coffee can be acquired by solving the heat equation, $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q=\rho c_p\frac{\partial T}{\partial t}$, where k is the thermal conductivity, q is the rate of heat generation/consumption from an internal source/drain, ρ is the density and C_p is the constant-pressure specific heat. Describe the physical meanings of the entire equation and of each term in the equation.

(b) (**3 points**) Following (a), what assumptions must be valid such that the heat equation holds true?

(c) (**3 points**) Assuming the heat dissipation from the top and bottom ends of the cup being negligible for the moment and the thermal properties of coffee being constant, simplify the heat equation from (a).

(d) (**3 points**) You may make an extra assumption(s) when carrying out the simplification in (c). Why is the assumption reasonable for the considered problem?

(e) (**11 points**) With the effective thermal conductivity, density and specific heat of coffee being 2.8 W/m·°C, 1100 kg/m^3 and 3400 J/kg·°C, respectively, for the water content of 70 wt%, exploit every boundary condition you can recognize and solve for the temperature distribution function of coffee immediately after it is poured into the cup.

(f) **(4 points)** With the inner diameter of the cup being 6 cm, use the temperature distribution function you obtained in (e) to determine the temperature of the inner surface of the cup.

When optimizing the wall thickness of the cup, you figure out that the outer surface temperature of the cup should be no higher than 35°C for consumers' comfort and safety when they hold the cup with a bare hand. Now the heat dissipation, via convection and radiation, from the lidless top of the cup cannot be ignored and needs to be taken into consideration along with the heat dissipation through the cup wall.

(g) **(11 points)** Assume that the heat dissipation is stable over the considered period of time and the coffee temperatures at the side and top surfaces are uniform. Given that the emissivity of coffee and the cup is 0.8, the cup measures 12 cm tall, the thermal conductivity of PLA is 0.193 W/m·°C and the room temperature is 25°C, along with the information in (f), determine the minimal wall thickness of the cup that meets the aforementioned criterion. (Hint: Ignore the surface area difference between the inner and outer surfaces of the cup. Due to the relatively small dimension of the wall thickness, the heat dissipation through the wall can be described in the context of the **plane wall** geometry. The consumers' preference depicted above should be taken into account.)

NEWTON'S LAW OF VISCOSITY

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + \left(\tfrac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-8)}^a$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + \left(\tfrac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-9)}^a$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + \left(\tfrac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v}) \qquad \text{(B.1-10)}^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad \text{(B.1-11)}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right] \qquad \text{(B.1-12)}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right] \qquad \text{(B.1-13)}$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad \text{(B.1-14)}$$

注意：背面有試題

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THE EQUATION OF CONTINUITY[a]

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical coordinates (r, θ, φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-6)}$$

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specic Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specic Heat (kJ/kg·K) $c_{p,f}$	$c_{p,g}$	Viscosity (N·s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m·K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683