國立中央大學 113 學年度碩士班考試入學試題

所別： 企業管理學系 碩士班 工商管理乙組(一般生)
企業管理學系 碩士班 工商管理丙組(一般生)

科目： 統計學

＊本科考試可使用計算器，廠牌、功能不拘
※作答題需計算過程，無計算過程者不予計分

1. Assume that there is a relationship between random variable Y_i and the corresponding given value X_i as following:

$$Y_i = \beta X_i + \varepsilon_i, \quad i = 1, 2, \ldots, n,$$

where β is an unknown parameter, and ε_i is independently and normally distributed with mean 0, and variance σ^2.

(a) Find the least squares estimator $\hat{\beta}$, such that $\sum_{i=1}^{n}(Y_i - \hat{\beta}X_i)^2$ minimized. (5 pts)

(b) Show that whether or not the least squares estimator $\hat{\beta}$ in (a) is the same as the maximum likelihood estimator. (5 pts)

(c) Given a particular value, x_p, what is the distribution of Y_p, where $Y_p = \beta x_p + \varepsilon_i$. (5 pts)

(d) Given a particular value, x_p, what is the distribution of $\hat{Y}_p$, where $\hat{Y}_p = \hat{\beta}x_p$. (5 pts)

(e) Construct an 100(1-α)% confidence interval of E(Y_p) (the expected value of Y_p). (5 pts)

2. Suppose the multivariate distribution of X_1 and X_2 is *uniform* over the region $\{0 \le x_1 \le 1, x_1 \le x_2 \le (x_1+1)\}$, i.e. the joint probability density function is as following,

$$f(x_1, x_2) = \frac{1}{k} \quad \text{if } 0 \le x_1 \le 1 \text{ and } x_1 \le x_2 \le (x_1+1), \text{ or}$$

$$= 0 \quad \text{o.w.}$$

(a) What is the value of k? (2pts)

(b) Show the marginal distribution of X_1, and the marginal distribution of X_2, respectively. (8 pts)

(c) Determine the correlation coefficient of X_1 and X_2. (5 pts)

(d) Given a value τ, $0 \le \tau \le 2$, let's define a payoff function, $g(x_1, x_2)$, as following,

$$g(x_1, x_2) = \tau \quad \text{if } x_2 \ge \tau, \text{ or}$$

$$= x_2 \quad \text{if } x_2 < \tau.$$

Find the expected payoff (representing as a function of τ). (5 pts)

(e) Find the variance of payoff in (d). (5 pts)

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3. In Taiwan, you will get an uniform invoice after making every purchase. Every two months, there will be an uniform invoice lottery, and you can check your collecting uniform-invoices with the winning numbers to see if you win any prizes. Suppose the winning/bingo rate of any uniform invoice is p. Let's use a random variable, N, to represent the number of your collecting uniform-invoices during the two months period. Assume the distribution of N is Poisson with mean 60.
   (a) Suppose the winning/bingo rate of any uniform invoice is $p = 0.005$, what is the probability that none of you collecting uniform invoices during these two months wins any prize? (5 pts)
   (b) Given $p = 0.005$, what is the distribution of the number of winning uniform-invoices? (7 pts)
   (c) Given $p = 0.005$, what is the expected number of winning uniform-invoices. (3 pts)
   (d) Suppose none of your collecting uniform-invoices last November and December won any prize, based on this experience, what is the maximal likelihood estimate of p? (5 pts)
   (e) Suppose during the whole year last year (6 times of lottery), the numbers of your winning uniform-invoices were 0, 1, 1, 0, 0, and 0. Based on these 6 lotteries, what is the maximal likelihood estimate of p? (5 pts)

4. Shewhart control chart is one of the statistical process control schemes, which uses "$\bar{x}$" as an indicator of whether the process is in control or out of control. For example, *temperature* is used to measure the output of a production process. When the process is in control, the mean of the process is $\mu = 200$ and the standard deviation is $\sigma = 0.5$.
   (a) If samples of size 6 are to be used, to monitor the process, please provide the LCL (lower control limit) and UCL (upper control limit) at 5% significance. (6 pts)
   (b) Is the process in control for a sample providing the following data? (2 pts)
   200.9 199.3 199.1 200.5 199.9 201.8
   (c) At the next sample point, the observed data are as following. Is the process still in control? (2 pts)
   200.9 200.5 200.7 199.3 200.1 198.2
   (d) At the next sample point following (c), the observed data are as following. Is the process still in control? (2 pts)
   200.7 200.6 200.5 199.2 200.0 198.1
   (e) Use the observed data in (b) and (c). Please conduct the test and determine whether the mean has changed or not? (4 pts)
   (f) Similarly, please conduct the test and determine whether the mean has changed or not between (c) and (d)? (4 pts)
   (g) Please provide your explanation about the contradiction (if you find any) for the in-control/out-of-control results in (b), (c), (d) and the mean-changed/mean-not-changed results in (e), (f). (5 pts)

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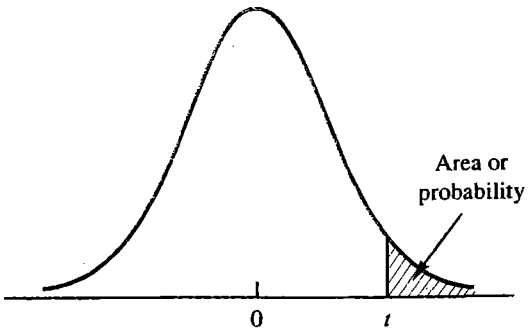

Entries in the table give *t* values for an area or probability in the upper tail of the *t* distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

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