科目 微積分 類組別 A2 共 2 頁 第 1 頁

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲）部分，共 8 題，命題型態為填充題，請依題號順序獨立列出，勿同時陳列出計算過程。倘若答案被包含在演算過程，將被視為試算流程，不另行挑出計分。

1. Find the limit $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.

2. Find $\frac{d^{2023}}{dx^{2023}}(x\sin x)$.

3. Evaluate the integral $\int_{-1}^{4}\frac{dx}{\sqrt{|x|}}$.

4. Evaluate the integral $\int_0^8\int_{\sqrt[3]{x}}^2\frac{1}{y^4+1}\,dydx$.

5. Consider the region bounded by the graphs of $y=\ln x$, $y=0$, and $x=e$. Find the volume of the solid formed by revolving the region about the x-axis.

6. Let $y=(\sin x)^x$, $\sin x>0$. Find $\frac{dy}{dx}$.

7. Assume that constants a and b are positive. Find equations for all horizontal asymptotes for the graph of $y=\frac{\sqrt{ax^2+4}}{x-b}$.

8. We say that the two commodities are **substitute commodities** if a decrease in the demand for one results in an increase in the demand for the other. Conversely, two commodities are referred to as **complementary commodities** if a decrease in the demand for one results in a decrease in the demand for the other as well. Suppose that the demand equations that relate the quantities demanded x and y to the unit prices p and q of the commodities A and B respectively are given by $x=f(p,q)=\frac{q^2}{q+p^2}$ and $y=g(p,q)=e^{-2q+p}$. Are A and B substitute, complementary or neither?

注意：背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Consider the function $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$.

   (a) (4 分) Show that f is continuous at $x = 0$.

   (b) (4 分) Find $f'(x)$ for $x \neq 0$.

   (c) (4 分) Use the limit definition of the derivative to show that f is not differentiable at $x = 0$.

2. (a) (6 分) Use the integral test to determine if the series $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$ converges or diverges.

   (b) (6 分) Find all values of x for which $\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right)(x-3)^n$ converges absolutely.

3. Suppose x units of labor and y units of capital are required to produce

$$f(x, y) = 100x^{3/4}y^{1/4}$$

units of a certain product. If each unit of labor costs \$200, each unit of capital costs \$300, and a total of \$60,000 is available for production, determine how many units of labor and how many units of capital should be used to maximize production.