類組：電機類　科目：工程數學A(3003)　　共 7 頁 第 1 頁

多重選擇題，共20題，每題5分 每題每一選項(ABCDE)單獨計分，每一選項個別分數為1分

答錯一個選項倒扣1分，倒扣至本大題(即多選題)0分為止。

1. Find the solution of $y' - y = 1$.
   (Note: C is the constant)

(A) $y = C \cdot e^x$
(B) $y = C \cdot e^x + 1$
(C) $y = C \cdot e^x - 1$
(D) $y = (C+1) \cdot e^x$
(E) $y = (C-1) \cdot e^x$

2. Find the solution of $(1 - x^2)y - xy' = 0$
   (Note: C is the constant)

(A) $y = C\sqrt{1 - x^2}$
(B) $y = \frac{C}{\sqrt{1-x^2}}$
(C) $y = -\frac{1}{2}x^3 + Cx$
(D) $y = Cxe^{-\frac{1}{2}x^2}$
(E) None of the above

3. Find the solution of $y'' = sin(-x)$
   (Note: C_1, C_2 are the constant)

(A) $y = sin(-x)$
(B) $y = -sin(-x)$
(C) $y = -sin(-x) + C_1x + C_2$
(D) $y = sin(-x) + C_1x + C_2$
(E) None of the above

4. Find the solution of $xy' + y = 3$
   (Note: C is the constant)

(A) $y = \frac{C}{x} + 3$　(C) $y = \frac{C}{x} - 3$　(E) None of the above
(B) $y = \frac{3}{x} + C$　(D) $y = \frac{x}{C} - 3$

注意：背面有試題

5. For which values of m is the function $y = x^m$, a solution of the differential equation?

$$x^2y'' - 5xy' + 8y = 0$$

(A) $m = 2, 3$
(B) $m = 2, 4$
(C) $m = 3, 4$
(D) $m = -2, -4$
(E) None of the above

6. In the Taylor's series expansion of e^{2x} about $x = 3$, the coefficient of $(x-3)^5$ is

(A) $1/5!$

(B) $3^5/5!$

(C) $16 \times e^6/6!$

(D) $32 \times e^6/5!$

(E) $64 \times e^6/4!$

7. Which of the following functions would have only even powers of x in its Taylor series expansion about the point $x = \pi/2$?

(A) $\sin(2x + \pi/4)$
(B) $\cos(3(x - \pi/2))$
(C) $\exp(x^2 - \pi)$
(D) $\frac{1}{(1+x^2-\pi)}$
(E) $\frac{x}{(1+x^3-\pi)}$

注意：背面有試題

8. Evaluate the following integral. $\int_{-\pi/2}^{\pi/2} \frac{1}{1+\sin^2\theta} d\theta$

(A) $\pi^4/4$

(B) $\pi^3/2\sqrt{2}$

(C) $\pi^2/2$

(D) $\pi/\sqrt{2}$

(E) $\sqrt{\pi}/\sqrt{8}$

9. Evaluate $\int_0^{\infty} \frac{dx}{1+x^{56}}$

(A) $\frac{\pi}{56\sin(\frac{\pi}{56})}$

(B) $\frac{\pi}{28\sin(\frac{\pi}{28})}$

(C) $\frac{\pi}{112\sin(\frac{\pi}{112})}$

(D) π

(E) 2π

10. Evaluate the following integral $\oint_C \frac{Ln(z+1)}{z^2+1} dz$ C: $|z-i| = 1.4$. Here Ln function denotes the principal part of natural logarithm function ln and is hence single-valued. What is the imaginary part of this integral?

(A) $\frac{\pi^2}{8}$

(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{8}$

(E) $\pi\ln(\sqrt{2})$

注意：背面有試題

11. Let $A=\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$, which of the following descriptions are true?

(A) A is invertible.

(B) A is diagonalizable with real-valued eigenvalues.

(C) A is normal.

(D) A is Hermitian.

(E) A is unitary.

12. Which of the following properties of the "determinant" of an n x n matrix are incorrect?

(A) We can use "cofactor expansion" to calculate the determinant along any row or column.

(B) For an invertible matrix, its determinant cannot be 0.

(C) For two n x n matrices: A and B, $\det(AB) = \det(A)\cdot\det(B)$.

(D) If we apply elementary row operations to calculate a matrix's determinant, it requires less multiplications than using cofactor expansion.

(E) For an upper triangular matrix, its determinant is the product of the diagonal elements.

13. Which of the following properties on subspace are correct?

(A) Every subspace contains infinite number of vectors.

(B) $\{0\}$ is a subspace of any vector space.

(C) The basis of a subspace can be extended to a basis of the vector space that contains this subspace.

(D) The dimension of a subspace is less than the vector space that contains this subspace.

(E) The interception of any two subspaces contains at least one vector.

14. For a linear transformation T: $R^3 \rightarrow R^3$ defined by: $\mathbf{T}(x,y,z) = (3x+2y,-2x+3y,5z)$, which of the following statements are correct?

(A) The basis of the kernel (null space) of T is $\{0\}$.

(B) T is one to one.

(C) T is onto.

(D) T is invertible.

(E) T is diagonalizable.

注意：背面有試題

15. For a 5x5 matrix: $A(t) = \begin{pmatrix} 6 & 1 & -2 & 0 & 0 \\ 2 & 2 & 3 & 2 & 2 \\ 4 & 3 & 1 & 3 & 4 \\ -1 & 3 & 2 & 0 & 0 \\ 2 & 1 & -1+\cos t & 1 & 2 \end{pmatrix}$, which value of t will make $\det(A) = 0$?

(A) $-\pi$
(B) $-\pi/2$
(C) 0
(D) $\pi/2$
(E) π

16. What are the corresponding eigen vectors for the matrix A having the following form,

$$A = \begin{pmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

(A) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$

(E) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

注意：背面有試題

17. Following the previous question, evaluate A^{2301}.

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$,

(C) $\begin{pmatrix} 1 & 2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$,

(D) $\begin{pmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$,

(E) $\begin{pmatrix} 1 & -2 & 0 \\ 0 & -1 & 8 \\ 0 & 0 & -1 \end{pmatrix}$,

18. For a rectangle matrix $M = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$, how to find the corresponding singular value?

(A) By finding the eigen-values of $M = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \\ 0 & 0 & 0 \end{pmatrix}$,

(B) By finding the eigen-values of $MM = M^2$

(C) By finding the eigen-values of $\lim_{n\to\infty} M^n$

(D) By finding the eigen-values of $\frac{M}{|M|}$

(E) By finding the eigen-values of MM^t

注意：背面有試題

19. Following the previous question, find the largest singular value of the matrix M

(A) 600

(B) 360

(C) $10\sqrt{6}$

(D) $6\sqrt{10}$

(E) $3\sqrt{6}$

20. Which of the following number β can make the matrix B being positive definite,

$$B = \begin{pmatrix} \beta & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \beta \end{pmatrix}$$

(A) -1

(B) 0

(C) 1

(D) 2

(E) i