科目 __微積分__　　類組別 ___A3/A4/A6___ 　　　　

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲）部分，共 8 題，命題型態為填充題，請依題號順序獨立列出，勿同時陳列出計算過程。倘若答案被包含在演算過程，將被視為試算流程，不另行挑出計分。

1. Evaluate the limit $\lim\limits_{x \to 2} \dfrac{1}{x-2} \displaystyle\int_4^{x^2} \dfrac{\sin t}{t} \, dt$.

2. Evaluate the integral $\displaystyle\int_0^\infty e^{-2x} \cos x \, dx$.

3. Evaluate the integral $\displaystyle\int_0^{\ln 17} \dfrac{e^x \sqrt{e^x - 1}}{e^x + 15} \, dx$.

4. Evaluate the double integral $\displaystyle\iint_D \left(x^2 + y^2\right)^{3/2} dA$, where D is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

5. Evaluate the double integral $\displaystyle\iint_R (x + y) e^{x^2 - y^2} \, dA$, where R is the rectangle enclosed by the line $x - y = 0$, $x - y = 2$, $x + y = 0$, and $x + y = 3$.

6. Evaluate the triple integral $\displaystyle\iiint_E x^2 \, dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 \le 4$, $y \ge 0$.

7. Find the length of the curve $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \ln(\cos t)\mathbf{k}$, $0 \le t \le \pi/4$.

8. Find the work done by the force field $\mathbf{F}(x, y) = x\mathbf{i} + (y + 2)\mathbf{j}$ in moving an object along an arch of the cycloid $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$, $0 \le t \le 2\pi$.

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. **(a)** (6 分) Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^{2n}(2n)!}$.

   Then find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!}$.

   (b) (6 分) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n(1+\ln^2 n)}$ is absolutely convergent, conditionally convergent, or divergent.

2. Find the limit or show that the limit does not exist.

   (a) (6 分) $\displaystyle\lim_{x\to(\frac{\pi}{2})^-} (\tan x)^{\cos x}$.

   (b) (6 分) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$.

3. Find the local maximum and minimum values and saddle point of the function

$$f(x,y) = \left(x^2 + y^2\right) e^{-x}.$$