

國立中央大學 113 學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

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科目： 微積分

* 本科考試禁用計算器

計算題應詳列計算過程，無計算過程者不予計分

Problem 1. (12%) Let a be a non-zero real number, $\lambda > 0$, and f be a function defined by

$$f(x) = \frac{1}{2}(x - a)^2 + \lambda|x|.$$

Show that f attains its absolute minimum at the point $x = \operatorname{sgn}(a) \max\{|a| - \lambda, 0\}$, where sgn is the sign function (so that $\operatorname{sgn}(2) = 1$ and $\operatorname{sgn}(-\sqrt{3}) = -1$, and etc). If you don't know how to proceed, you can try specific a and λ first.

Problem 2. (12%) Find $\frac{d}{dx} \int_{\ln(x^2+1)}^{\arctan(x^2)} \exp(t^2) dt$.

Problem 3. (12%) Find the indefinite integral $\int \frac{dx}{x^6(x^2+1)}$.

Problem 4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} (1+x)^{\frac{1}{x}} & \text{if } x \neq 0, \\ e & \text{if } x = 0. \end{cases}$$

1. (10%) Find the first three terms of the Maclaurine series of f ; that is, find a_0, a_1, a_2 so that

$$f(x) = a_0 + a_1x + a_2x^2 + O(x^3) \quad \text{as } x \rightarrow 0$$

or equivalently (in this case),

$$\lim_{x \rightarrow 0} \frac{f(x) - a_0 - a_1x - a_2x^2}{x^3} \text{ exists.}$$

2. (10%) Find the limit

$$\lim_{x \rightarrow \infty} x \left[\left(1 + \frac{1}{x}\right)^x - e \right].$$

Problem 5. (10%) Discuss the continuity of the function $f(x, y) = \begin{cases} \frac{\sin(xy)}{xy} & \text{if } xy \neq 0, \\ 1 & \text{if } xy = 0. \end{cases}$

Problem 6. (12%) Find the maximum value of the function $f(x, y) = x^2ye^{-x^2-y^2}$.

Problem 7. (12%) Use the method of Lagrange multiplier to maximize the function $f(x, y, z) = yz + xy$ subject to the constraint $xy = 1$ and $y^2 + z^2 = 1$.

Problem 8. (10%) Find the double integral $\iint_R xy \, dA$, where R is the region bounded by the line $y = x - 1$ and parabola $y^2 = 2x + 6$.