國立中央大學 114 學年度碩士班考試入學試題

系所：　地球科學學系地球物理 碩士班 不分組(一般生)
　　　　地球科學學系地球物理 碩士班 不分組(在職生)

科目：　微積分

＊本科考試禁用計算器

作答時須列出完整計算過程

1. (a) (5%) $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) =?$

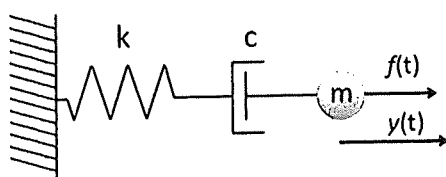   (b) (5%) $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x =?$　　[hint: $\ln(1+u) = u$, for small u]

2. (a) (5%) $\frac{d}{dx}\left(\int_{A(x)}^{B(x)} e^t dt\right) =?$

   (b) (5%) $\frac{df(x)}{dx} =?$　$f(x) = \sqrt{x^2 + 1}$

3. (a) (5%) $\int x\ln(x)dx = ?$

   (b) (5%) $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$

4. (a) (5%) Derive the ordinary differential equation for the mass-spring-damper system as shown in the figure, where k is the spring constant, m is the mass of the ball, c is the damping constant, $y(t)$ is position of the ball, and $f(t)$ is the input force.

   (b) (5%) Solve for $y(t)$ in the case where c = 0 and $f(t) = 0$.

注意:背面有試題

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5. (10%) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$.

6. (10%) Derive the relationship between the half life $T_{1/2}$ of a radioactive substance and its decay constant λ, given that the decaying rate is proportional to its remaining quantity.

7. (10%) Using triple integrals, calculate the volume of a cone with base radius a and height h.

8. (a) (4%) What is Heaviside function $u(t - a)$ and find its Laplace Transform $\mathcal{L}\{u(t - a)\}$?
   (b) (6%) Find the inverse transform $f(t)$ of
   $$F(s) = \frac{e^{-2s}}{s^2 + \pi^2} + \frac{e^{-2s}}{(s + 2)^2}$$

9. (10%) Use the method of separating variables to solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, for the temperature of a metal bar of length L.
   The boundary conditions are $u(0, t) = 0$, $u(L, t) = 0$ for all t.
   The initial conditions are $u(x, 0) = f(x)$.

10. (10%) Find the **odd** periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \dfrac{2k}{L}x & \text{if} \quad 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L - x) & \text{if} \quad \dfrac{L}{2} < x < L. \end{cases}$$

注意:背面有試題