所別：**大氣物理研究所碩士班　不分組** 科目：**應用數學**

1. If the isotherms (=curves of constant temperature) in a body are
    $$T(x, y) = 2x^2 + y^2 = \text{constant}$$
    What are their orthogonal trajectories (the curves along which heat will flow in regions free of heat sources or sinks and filled with homogeneous material)?
    (10%)

2. What is the superposition principle (linearity principle)?　Does it hold for nonlinear equations?　For nonhomogeneous linear equations?　For homogeneous linear equations?　Why is it important?
    (10%)

3. Find a general solution of the following nonhomogeneous equation.
    $$y'' + 10y' + 25y = e^{-5x}$$
    (10%)

4. Find the eigenvalues and eigenfunctions of the following Sturm-Liouville problem.
    $$y'' + \lambda y = 0, \qquad y(0) = 0 \text{ and } y'(L) = 0$$
    (10%)

5. Find the Laplace transform of the following function.
    $$te^{-t} \sin t$$
    (10%)

6. Diagonalize the following matrix.
    $$\begin{bmatrix} -8 & 11 & 3 \\ 4 & -1 & 3 \\ -4 & 10 & 6 \end{bmatrix}$$
    (10%)

7. Why did we sometimes use polar or spherical coordinates?　Why and where did Bessel's equation occur?　Legendre's equation?　Write down the Laplace equation in cylindrical and spherical coordinates.
    (10%)

8. (a) Please write down the definitions of curvature, gradient, divergence, and curl, and identify which of them are scalar or vector.
    (b) For each of the above quantities, use a physical phenomenon to explain the meaning of it.
    (10%)

9. If $\vec{F} = \nabla \times \vec{A}$, and S represents the surface of a sphere, with radius R.　Please compute the surface integration of $\vec{F}$ over S.
    (10%)

10. Let $\vec{F} = 2xy\,\vec{i} + x^2\,\vec{j}$.　If $\vec{F}$ is integrated along curve $y = x^2 + 1$ from (1, 2) to (3, 10), please compute the result of this line integration.
    (10%)

參考用