國立中央大學98學年度碩士班考試入學試題卷

所別：太空科學研究所碩士班 一般生　　科目：應用數學　　共 1 頁 第 1 頁

學位在職生　　＊請在試卷答案卷（卡）內作答

(1, 20%) Find a general solution of the equations:

(a) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 0$ (10%)　　(b) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$ (10%).

(2, 20%) Derive the following formulas involving del operators ($\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$):

(a) $\nabla \times (\nabla U) = 0$ (10%)　　(b) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ (10%),

where U is the scalar function and $\mathbf{A}$ the vector function of x, y, and z, and has partial derivatives.

(3, 15%) Suppose that $f(t)$ is continuous for all $t \geq 0$ and has a derivative $\frac{df(t)}{dt}$ that is piece-wise continuous on every finite interval in the range $t \geq 0$. Please derive the Laplace transform of the derivative to be expressed as the follow terms:

$$L\left[\frac{df(t)}{dt}\right] = sL[f(t)] - f(0).$$

Note that the Laplace transform of a function $f(t)$ is defined as $L[f(t)] \equiv \int_0^\infty f(t)e^{-st}dt$.

(4, 15%) The convolution $f * g$ of functions $f(t)$ and $g(t)$ is defined by

$$f * g \equiv \int_{-\infty}^{\infty} f(p)g(t-p)dp = \int_{-\infty}^{\infty} f(t-p)g(p)dp.$$

Let $f(t)$ and $g(t)$ be piece-wise continuous, bounded and absolutely integrable on the t-axis. Then derive that the Fourier transform of the convolution of the functions $f(t)$ and $g(t)$ to be

$$F[f * g] = \sqrt{2\pi}F[f(t)]F[g(t)].$$

Note that the Fourier transform of a function $f(t)$ is defined as $F[f(t)] \equiv \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$.

(5, 10%) Find the Taylor series of the function, $f(x) = a^x$, with a power series in powers of x (Maclaurin series) at least 4 terms. It is noted that the a is a constant.

(6, 20%) The error function is defined as $erf(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$. Evaluate the following value and derivative:

(a) $erf(\infty)$ (10%)　　(b) $\frac{d}{dx}[erf(x)]$ (10%).

參考用