國立中央大學八十四學年度碩士班研究生入學試題卷

所別：數學研究所 組 科目：高等微積分 共 1 頁 第 1 頁

參考用

1. For nonnegative integer n and real number x, let
$$f_n(x) = \sum_{m=0}^{n} \frac{x^2}{(1+x^2)^m}.$$
Prove or disprove that the sequence of functions $\{f_n\}$ converges uniformly in the real line $\mathbb{R}$ to a function f. (20%)

2. Let f be a complex-valued continuous function on $[0,\infty)$ and $\lim_{t\to\infty} |f(t)| = \infty$. Show that the range $I = \{f(t) ; t \in [0,\infty)\}$ intersects the boundary of every rectangle E containing $f(0)$. (20%)

3. Let a and b be positive real numbers such that $\frac{1}{a} + \frac{1}{b} = 1$. If $m \geq 0$ and $n \geq 0$, then $mn \leq \frac{m^a}{a} + \frac{n^b}{b}$. (20%)

4. Let $\mathbb{C}$ be the set of all complex numbers and $r > 0$. Suppose that f is a complex-valued differentiable function in $D = \{z ; z \in \mathbb{C} \text{ and } 0 < |z| < r\}$ and satisfies $|f(z)| \leq \frac{1}{\sqrt{|z|}}$ in D. Prove that f is bounded in D. (20%)

5. Give a function f which is defined in $\mathbb{R}$ and it has derivatives of all orders in $\mathbb{R}$, but
$$f(x) \neq \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!}(x-3)^n \text{ for some } x \in \mathbb{R}.$$ (20%)