

系所別: 工業管理研究所 乙組 科目:

作業研究

1. Consider the following primal problem

$$\begin{aligned} \max z &= \sum_{j=1}^n c_j x_j \\ \sum_{j=1}^n a_{ij} x_j &\leq b_i, i = 1, 2, \dots, m \\ x_1, x_2, \dots, x_n &\geq 0. \end{aligned}$$

It is known that its dual is

$$\begin{aligned} \min y_0 &= \sum_{i=1}^m b_i y_i \\ \sum_{i=1}^m a_{ij} y_i &\geq c_j, j = 1, 2, \dots, n \\ y_1, y_2, \dots, y_m &\geq 0. \end{aligned}$$

- (a) (10%) Consider the following primal problem

$$\begin{aligned} \max z &= x_{n+1} \\ \sum_{j=1}^n R_{ij} x_j &\geq x_{n+1}, i = 1, 2, \dots, m \\ x_1, x_2, \dots, x_{n+1} &\in R. \end{aligned}$$

Derive its dual by only using the above given primal-dual relation (Let y_1, y_2, \dots be the dual variable).

- (b) (5%) Using the dual problem, find the necessary and sufficient condition that the optimal solution in the primal problem is $x_1 = x_2 = \dots = x_{n+1} = 0$.
- (c) (5%) Using the dual problem, find the conditions on the dual problem such that the primal problem is unbounded to positive infinity.
- (d) (5%) In financial theory, we usually treat R_{ij} as the net profit obtained for each unit of asset j when the market is in state i . That $x_i > 0$ means that we buy x_i units of asset i and that $x_i < 0$ means that we sell x_i units of asset i . Explain the meanings of our primal problem in (a) and the results in (b) and (c).
2. Consider a n -month production planning problem in which inventory and backorder are allowed. For month i , let d_i be its demand to meet; u_i be its production capacity (upper bound); c_i be the production cost for each unit; a_i be the inventory cost for one unit; b_i be the backorder carrying cost for each backorder. We want to minimize the total cost (production cost plus inventory cost plus backorder carrying cost...).
- (a) (5%) Define your decision variables.
- (b) (10%) Formulate it as a minimum cost flow problem by drawing the corresponding network figure.
3. A player will play three consecutive matches. In each match, he can decide not to play or bet only 1 dollar on either Game A or Game B. The returns for each dollar you bet on Game A and Game B with the corresponding probabilities are as follows:

注意：背面有試題

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Game	Return(dollars)	Probability
A	0	0.3
	2	0.7
B	1	0.8
	2	0.2

Note that the return consists the money that he wins or loses and the money he bets. For example, return of \$2 means that he win \$1 in additional to the one dollar he bets. Given that initially he has one dollars,

- (a) (10%) Use Dynamic programming to find the optimal policy maximizing the probability that he will have at least 3 dollars after these three matches.
4. A new bank called OR bank now wants to hire several tellers for the future business. The arrival process of customers is estimated as a Poisson process with rate of 4 per minute. The service time for each customer in the counter is estimated as an exponential random variable with mean 1 minute. The service qualities are
- The average number of customers waiting in queue should be less than or equal to 2.
 - At least 95% of time that the number of customers waiting in queue should not excess 4.
 - The utilization of each server should not be less than 0.6.

Please answer the following questions.

- (a) (10%) Derive the limiting probability of the number of customers in system when there are n servers. What is the utilization?
- (b) (10%) Determine the minimum number of tellers needed for fulfilling the above three requirements if it exists.
5. A small parking lot in front a fast food restaurant has four parking spaces. A car that arrives finding parking space full will go away and never come back. The arrival process of the cars is a Poisson process with rate 20 per hour and each car occupies a parking space for an exponential time with mean 5 minutes. We would like to analyze the system by using queueing model.
- (10%) What is the loss probability of an arrival? What is effective arrival rate?
 - (10%) What is the average number of occupied parking spaces? What is the utilization of each parking space?
 - (10%) The whole parking space is rented by this restaurant and this restaurant has to pay park fee \$40 per hour per car and each car will spend \$200 for food. What is the total income for this restaurant per hour?