

1) 請找出以下邊界值問題的 eigenvalues 及 eigenfunctions:

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = y(2) = 0,$$

其中 λ 為 eigenvalue.

(25分)

2) 請找出以下方程的通解 (general solution):

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x$$

(25分)

3) Let $f(x, y) = \begin{cases} (x^2 + y^2) \sin \frac{1}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

(a) Calculate $f_x(0, 0)$ and $f_y(0, 0)$.

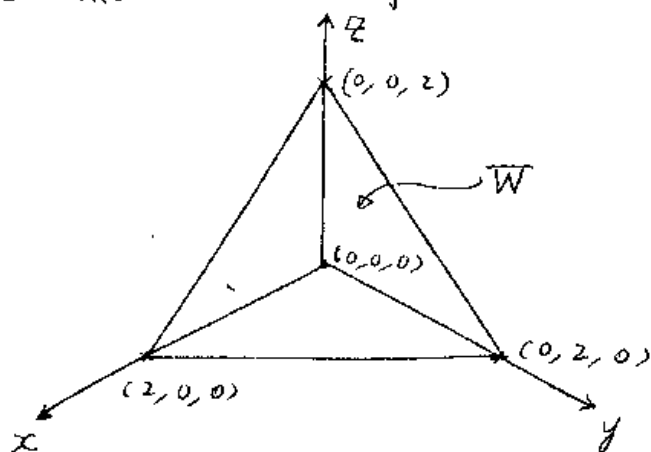
(b) Explain why f_x is not continuous at $(0, 0)$. (10分)

4) Given the space curve $x = t, y = t^2, z = \frac{2}{3} t^3$.

Find (i) the curvature K ,

(ii) the torsion τ . (20分)

5) Evaluate the volume of W by using divergence theorem.



(20分)