## 台灣聯合大學系統 105 學年度碩士班招生考試試題

類組: <u>電機類</u> 科目: <u>工程數學 D(3006)</u>

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### ※請在答案卷內作答

1. (15 points) Let Ax=b, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ -1 & -2 & 1 & 2 & 3 \\ 2 & 4 & -3 & 2 & 0 \\ -3 & -6 & 2 & 0 & 3 \end{bmatrix} \quad and \quad \mathbf{b} = \begin{bmatrix} 2 \\ 6 \\ 3 \\ 9 \end{bmatrix}$$

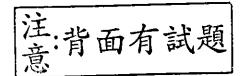
- (a) (7 points) Obtain the reduced row echelon form (RREF) of [A b].
- (b) (2 points) Obtain the rank and nullity of A.
- (c) (2 points) Are the columns of A linearly independent? Explain your answer.
- (d) (4 points) How many solutions for **Ax=b**? If it has solutions, write down the general solutions.
- 2. (15 points) Given the following matrix:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 & 2 & 1 \\ -2 & 2 & -1 & 1 & 4 \\ 4 & 1 & 15 & 12 & 19 \\ 6 & -6 & 9 & -4 & 0 \\ 4 & -2 & 9 & 2 & 9 \end{bmatrix}$$

- (a) (7 points) Obtain the determinant of A.
- (b) (2 points) Is A invertible? Explain your answer.
- (c) (4 points) Obtain the determinant of the following matrix.

$$\mathbf{B} = \begin{bmatrix} 4 & 1 & 15 & 12 & 19 \\ -2 & 2 & -1 & 1 & 4 \\ 2 & -1 & 3 & 2 & 1 \\ 4 & -2 & 9 & 2 & 9 \\ 6 & -6 & 9 & -4 & 0 \end{bmatrix}$$

(d) (2 points) Obtain the determinant of the inverse of B





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### ※請在答案卷內作答

3. (20 points) Given the following vector set

$$B = \{b_1, b_2, b_3\} = \left\{ \begin{bmatrix} 1\\1\\0 \end{bmatrix}, \begin{bmatrix} 1\\1\\1 \end{bmatrix}, \begin{bmatrix} 3\\2\\1 \end{bmatrix} \right\}$$

- (a) (8 points) Use the Gram-Schmidt process to generate a new orthonormal basis  $V=\{v_1, v_2, v_3\}$  from B (Please use the order  $b_1$ ,  $b_2$  and  $b_3$  to generate the new basis V).
- (b) (4 points) Use the new basis V to represent the following vector u:

$$\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

- (c) (4 points) Let  $\mathbf{w}$  be the orthogonal projection of  $\mathbf{b}_3$  to the plane spanned by  $\{\mathbf{b}_1, \ \mathbf{b}_2\}$ . Find  $\mathbf{w}$ .
- (d) (4 points) Find the orthogonal complement of  $\{b_1, b_2\}$ .

4. (16 points) Let 
$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x^2 & 0 \le x < \pi. \end{cases}$$

- (a) (8 points) Find the Fourier series of f(x) on the given interval.
- (b) (8 points) Use the result you obtained in part (a) to show that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$
 and  $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ 



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※請在答案卷內作答

#### 5. (10 points)

- (a) (5 points) Suppose that  $y = 1 x + 6x^2 + 3e^x + 5xe^x$  is a solution of a homogeneous fifth-order linear differential equation with constant coefficients. What is the corresponding auxiliary (or characteristic) equation of the differential equation?
- (b) (5 points) Please explain why the function  $F(s) = \frac{s^2 + 5s + 7}{s^2 + 2s + 4}$  is not the Laplace transform of a function that is piecewise continuous and of exponential order.
- 6. (24 points) Solve the following differential equations:

(a) (8 points) 
$$y'(t) = 1 - \sin t - \int_0^t y(\tau) d\tau$$
,  $y(0) = 0$ .

(b) (8 points) 
$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix}$$

(c) (8 points) 
$$(e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0$$
.

