## 國立中央大學 105 學年度碩士班考試入學試題

所別: 機械工程學系碩士班 熱流組(一般生)

共2頁 第1頁

機械工程學系光機電工程 碩士班 光機組(一般生)

能源工程研究所 碩士班 不分組(一般生)

科目: 工程數學

本科考試可使用計算器,廠牌、功能不拘

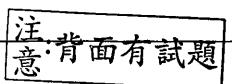
\*請在答案卷(卡)內作答

- 1. Solutions for ordinary differential equations (ODEs) (25%)
  - (a) Find the solution for the ODE y'+xy=x/y with the initial condition of y(0)=3 (5%)
  - (b) Find the solution for the general solution for the ODE  $y'' + 4y' + 4y = e^{-x} \cos x$  (5%)
  - (c) Solve the nonhomogeneous linear ODE  $x^2y'' xy' 3y = x^2$  by variation of parameters or undetermined coefficients. (5%)
  - (d) Solve the ODE  $(D^3 3D^2 + 3D I)y = 4\cos x$ . (5%)
  - (e)  $\begin{cases} y_1' = y_1 + y_2 + 10\cos x \\ y_2' = 3y_1 y_2 10\sin x \end{cases}$ , find  $y_1$  and  $y_2 = ?$  (5%)
- 2.

Given an ODE:  $y'' + 2y' - 3y = u(t-1) * \frac{1}{t^2 + 1} \delta(t-1)$ , where u(t) is unit step function,  $\delta(t)$  is impulse function, symbol \* denotes convolution,  $L(y) = Y(s) = \int_0^\infty y(t)e^{-st}dt$ , y(0) = y'(0) = 0.

- (a) Calculate  $L[u(t-1)*\frac{1}{2}\delta(t-1)] = ?$  (10%)
- (b) Find the solution  $y(t) = L^{-1}[Y(s)]$  of the above ODE. (15%)
- 3.
- (1) Evaluate the surface integral  $\int_{A} (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{n}dA$  over a unit cube. The cube has the volume center located at the origin (0, 0, 0), and its six plane surfaces are perpendicular to the x-, y-, and z- axes respectively, and  $\hat{n}$  is the unit normal vector pointing outwards from the surface.

  (5%)
- (2) Evaluate the contour integral  $\oint x dy y dx$  around the following closed curves in the x-y plane.
  - (a) The curve comprises four sides of a unit square whose center is located at the origin (0, 0). (5%)
  - (b) The curve is the circumference of a unit circle whose center is located at the origin (0, 0). (5%)
- (3) Consider the matrix  $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ 
  - (a) Find the eigenvalues and eigenvectors of A. (5%)
  - (b) Find the eigenvalues and eigenvectors of A<sup>5</sup>. (5%)



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共乙頁 第乙頁

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4. Partial differential equations (PDEs)

(a) Use separation of variables to find the solution of 
$$y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 0$$
 (10%)

(b) Solve the boundary value-problem  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -h$ , where h is a positive constant, h > 0. The

boundary conditions are given as: 
$$\begin{cases} u(0, y) = 0, \ u(\pi, y) = 1, \ y > 0 \\ u(x, y) = 0 \text{ as } y \to \infty \\ u(x, 0) = 0, \ 0 < x < \pi \end{cases}$$
 (15%)

注:背面有試題