國立中央大學106學年度碩士班考試入學試題

所別： 太空科學研究所 碩士班 不分組(一般生)　　共1頁 第1頁

太空科學研究所 碩士班 不分組(在職生)

科目： 應用數學

本科考試禁用計算器　　＊請在答案卷內作答

請注意：作答時，請寫出推導計算步驟或用文字說明如何獲得答案。如果只列出最後答案，卻沒有推導計算步驟或文字說明，該題將不予計分。

(1, 20%) Find a general solution of $y'=(x+4x)^2$.

(2, 20%) Find a general solution of the Euler-Cauchy equation. Show the details of your work.

$x^2y''-3xy'+4y=0$.

參考用

(3, 20%) Please find the following formulas of $J_\nu(x)$ with $J_{\nu-1}(x)$ and $J_{\nu+1}(x)$:

(a) $\dfrac{d\left[x^\nu J_\nu(x)\right]}{dx}=x^\nu J_{\nu-1}(x)$ and $\dfrac{d\left[x^{-\nu}J_\nu(x)\right]}{dx}=-x^{-\nu}J_{\nu+1}(x)$ (10%),

(b) $\dfrac{2\nu}{x}J_\nu(x)=J_{\nu-1}(x)+J_{\nu+1}(x)$ and $2\dfrac{dJ_\nu(x)}{dx}=J_{\nu-1}(x)-J_{\nu+1}(x)$ (10%).

It is noted that $J_\nu(x)=x^\nu\sum_{m=0}^{\infty}\dfrac{(-1)^m x^{2m}}{2^{2m+\nu}m!\Gamma(\nu+m+1)}$.

(4, 20%) Given matrix $\mathbf{M}=\begin{bmatrix}1&0&-3\\0&1&0\\-2&0&1\end{bmatrix}$ and column vector $\mathbf{v}=\begin{bmatrix}1\\2\\3\end{bmatrix}$. Let $\mathbf{u}=\mathbf{M}^{99}\mathbf{v}$. Determine the column vector $\mathbf{u}$.

(5, 20%) Find the temperature, $T(x,t)$, for a long thin metal bar in a model of the one-dimensional heat equation, $\dfrac{\partial T(x,t)}{\partial t}=c^2\dfrac{\partial^2 T(x,t)}{\partial x^2}$, where c is constant in this case. The bar at the ends $x=-L$ and $x=L$ are kept at temperature zero, $T(-L,t)=0$ and $T(L,t)=0$ for all $t\geq 0$. The initial temperature in the bar at time $t=0$ is given as $T(x,0)=f(x)$ for $-L\leq x\leq L$.