所別： 光電科學與工程學系 碩士班 不分組(一般生)　　　共 **2** 頁　第 **1** 頁

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　　　　＊請在答案卷　內作答

須有計算過程

1.(a) Find a Fourier Sine-series which represents the following function:

$$f(x) = 1 + \operatorname{Sin}(\pi x/2) \qquad 0 < x < 2$$

in the range [0, 2].

(b) Find the value of the sum of your series at x = 0 and x = 2. Are they equal to

f (x) at x = 0 and x = 2, respectively? If not, why? (25%)

2. Consider the following differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(x,y) = (\text{constant}) f(x,y)$$

in the rectangular region:　$0 \leq x \leq a$

$0 \leq y \leq b$

with the following boundary conditions: the normal derivatives of $f(x,y)$ at the boundary of the region specified above are zero. (a) Determine the right kind of constant in the equation so that it admits a solution. (b) Find the particular (25%) solutions of the differential equation for the constant you have obtained in part-a.

3. (a) The vector field $\vec{F} = \hat{x}(c_1 z + 3) + \hat{y}(y + c_2 z) + \hat{z}(3z + 2x)$ is said to be non-rotational. Find the constants c_1 and c_2. Note that $\hat{x}, \hat{y}$, and $\hat{z}$ are unit vectors in the x, y, and z directions. (5%)

(b) From the gradient of what scalar function can we obtain this vector field? (5%)

4. Find the solution for the differential equation $\dfrac{d^2 y}{dt^2} + 3\dfrac{dy}{dt} + 2y = \cos(t)$, which satisfies the initial condition $y(0) = 1$ and $y'(0) = 0$. We note that y' means $\dfrac{dy}{dt}$. (13%)

注意：背面有試題

參考用

5. Find the eigenvalues and the eigenfunctions for the **inverse matrix** of matrix **A**,

where $A = \begin{bmatrix} -2 & 1 \\ \dfrac{3}{2} & -\dfrac{1}{2} \end{bmatrix}$．　　(13%)

6. Find the Laplace Transform of $\sin\sqrt{t}$．　Or, put in mathematical form, find

$\mathcal{L}\{\sin\sqrt{t}\} = ?$　　(14%)