國立中央大學 106 學年度碩士班考試入學試題

所別: 數學系碩士班 應用數學組(一般生)

共一頁 第一頁

科目: 微積分

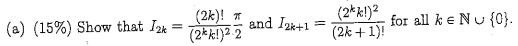
本科考試禁用計算器 須有計算過程

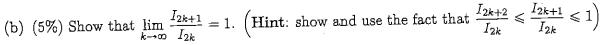
*請在答案卷

內作答

Problem 1. (12%) Find $\frac{d}{dx} \int_{2^x}^{\cos^{-1} x} \ln(u^4 + 1) du$.

Problem 2. Let $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$.





(c) (8%) Use (b) to find the limit $\lim_{k\to\infty}\frac{k!e^k}{k^{k+0.5}}$

Problem 3. (15%) Find the indefinite integral $\int \frac{1}{2 + \cos x} dx$.

Problem 4. (10%) Find the Maclaurin series of the the function $f(x) = \tan^{-1}(x^2)$. Note that you need to specify the interval of convergence of the Maclaurin series.

Problem 5. (13%) Find all $p \in \mathbb{R}$ such that the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\ln n)^p}{n}$ is convergent.

Problem 6. (12%) Find all relative extrema and saddle points of the function $F(x,y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$.

Problem 7. (10%) Let w = f(x, y, z) and $g(r, \theta, z) = (r \cos \theta, r \sin \theta, z)$. Show that

$$(w_x^2 + w_y^2 + w_z^2) \circ g = \left| \frac{\partial (w \circ g)}{\partial r} \right|^2 + \frac{1}{r} \left| \frac{\partial (w \circ g)}{\partial \theta} \right|^2 + \left| \frac{\partial (w \circ g)}{\partial z} \right|^2,$$

where w_x , w_y , w_z denote the partial derivative of w with respect to x, y, z, respectively.