＊請在答案卷內作答

Please show details of your work

1. (10%) Solve the initial value problem

$$y'' + 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -5$$

2. (10%) Find the odd periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if} \quad 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if} \quad \frac{L}{2} < x < L. \end{cases}$$

參考用

3. (10%) Find the Fourier transform of e^{-ax^2}, where $a > 0$.

4.

(10%) Use the method of separating variables to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L. The bounday conditions are $u(0,t) = 0$ and $u(L,t) = 0$ for all t. The initial conditions are $u(x,0) = f(x)$ and $\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = g(x)$. Make sure to dicuss the cases with $(\text{i})\, k = 0$, $(\text{ii})\, k > 0$, and $(\text{iii})\, k < 0$, where k is the assigned constant.

5. (10%) Find curl $\vec{v}$, where $\vec{v} = \left[ln(x^2 + y^2),\ 2tan^{-1}\left(\frac{y}{x}\right),\ 0\right]$ is given with respect to right-handed Cartesian coordinates.

6. (10%) Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

注意:背面有試題

7. (10%) (a) Solve $\nabla^2 u = 0$ in spherical coordinates with the boundary conditions $u(R,\ \varphi) = f(\varphi)$ *and* $lim_{r\to\infty} u(r,\ \varphi) = 0$. Find *u* inside and outside *r=R*.
(b) Discuss the spatial patterns (nodal lines) of its eigenfunctions such as u_3.

8. (10%)

Use Laplace transform to solve $\dfrac{\partial^2 w(x,t)}{\partial t^2} = c^2 \dfrac{\partial^2 w(x,t)}{\partial x^2}$,

with B.C. $\Rightarrow w(0,\ t) = f(t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$;

$\lim_{x\to\infty} w(x,\ t) = 0 \qquad (t \ge 0)$

I.C. $\Rightarrow w(x,\ 0) = 0, \quad \left.\dfrac{\partial w}{\partial t}\right|_{t=0} = 0.$

參考用

9. (10%) The Gamma function is defined as $\Gamma(v) = \int_0^\infty e^{-t} t^{v-1} dt$.

Show that $\Gamma(v+1) = v\Gamma(v)$ and $\Gamma(1) = 1$.

10. (2%) (a) Define Dirac's delta function $\delta(t-a)$.
(3%) (b) Derive Laplace transform of $\delta(t-a)$.
(5%) (c) Using Laplace transform to solve

$$y'' + 3y' + 2y = \delta(t-1)$$
$$y(0) = 0, \qquad y'(0) = 0$$