

甲、簡答題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為簡答題，請勿列出計算過程，倘若答案被包含在演算過程，將被視為試算流程，不予另行挑出計分。

1. Find the limit: $\lim_{x \rightarrow \infty} \frac{x + \cos x}{x - \cos x}$.

Answer : _____

2. Find the smallest positive ($x > 0$) inflection point of $F(x) = \int_0^x \cos(t^{3/2}) dt$.

Answer : _____

3. How many local extreme values does the function $f(x, y) = 10xye^{-(x^2+y^2)}$ have?

Answer : _____

4. Let C be the curve of intersection of the two surfaces $x^2 + y^2 + z^2 = 3$ and $(x - 2)^2 + (y - 2)^2 + z^2 = 3$. Find parametric equations of the tangent line to C at $P = (1, 1, 1)$.

Answer : _____

5. Evaluate $\iint_R \ln \sqrt{x^2 + y^2} dA$ where R is the unit disk $x^2 + y^2 \leq 1$.

Answer : _____

6. Find the volume between the two spheres: $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 2$ and inside the cone $z^2 = x^2 + y^2$.

Answer : _____

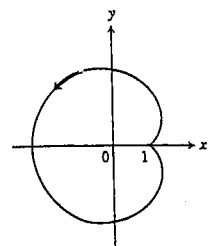
7. Calculate $\iint_R e^{9x^2+4y^2} dA$ where R is the interior of the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 \leq 1$.

Answer : _____

8. Find the area of the region enclosed by the limaçon

$$x = 2 \cos t - \cos 2t, \quad y = 2 \sin t - \sin 2t, \quad 0 \leq t \leq 2\pi$$

Answer : _____



注意：背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Let

$$f(x, y) = \begin{cases} \frac{(xy)^p}{x^4 + y^4}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

Use polar coordinates to show that $f(x, y)$ is continuous at all (x, y) if $p > 2$, but discontinuous at $(0, 0)$ if $p \leq 2$.

2.

a. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$ diverges or converges conditionally or converges absolutely and give reasons for your answer. (6 points)

b. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \left(\frac{3 + \sin(a_n)}{5}\right)^n$ converges. (6 points)

3. Goods 1 and 2 are available at prices (in dollars) of p_1 per unit of good 1 and p_2 per unit of good 2. A utility function $U(x_1, x_2)$ is a function representing the utility or benefit of consuming x_j units of good j . The marginal utility of the j th good is $\partial U / \partial x_j$, the rate of increase in utility per unit increase in the j th good. Prove the following law of economics: Given a budget of L dollars, utility is maximized at the consumption level (a, b) where the ratio of marginal utility is equal to the ratio of prices:

$$\frac{\text{Marginal utility of good 1}}{\text{Marginal utility of good 2}} = \frac{\partial U / \partial x_1}{\partial U / \partial x_2} = \frac{p_1}{p_2}$$