# 國立中央大學 113 學年度碩士班考試入學試題

所別： 財務金融學系 碩士班 甲組(一般生)　　第 1 頁 / 共 4 頁

科目： 統計

＊本科考試禁用計算器

**Present all your calculations, ensuring a clear demonstration of the methodologies employed. Your evaluation will consider the accuracy and completeness of both your methods and the results, along with the clarity of your explanations. If the conditions are not sufficient to answer the questions, please clarify and make your own ones while answering.**

1. You have a dataset of per capita $CO_2$ emissions ($y$) and per capita GDPs ($x$) from 50 countries in 2023. You run a regression of $y$ on $x$ and $x^2$, and obtain the following estimated results

   $\hat{y} = 50 + 0.12\,x + 0.0007\,x^2$
   (30.3) (0.05) (0.0002)

   where standard errors are reported in parentheses.

   (a) Is it necessary to assume that the error terms of the regression model are normally distributed? Explain. (3%)

   (b) What is the marginal effect of per capita GDP on per capita $CO_2$ emission? (4%)

   (c) Would you keep the quadratic term ($x^2$) in the model? Explain. (4%)

   (d) Do you think this regression model provides an unbiased estimator for the ceteris paribus effect of per capita GDP on per capita $CO_2$ emission conditional on a certain level of per capita GDP? (4%)

2. Let $E(y) = \mu_y$, and $E(x) = \mu_x$. Consider two regression models:

   Model A: $y = \beta_0 + \beta_1 x + \varepsilon$,

   Model B: $y^* = \gamma_0 + \gamma_1 x^* + u$, where $y^* = y - \mu_y$, $x^* = x - \mu_x$

   Assume $E(\varepsilon) = 0$. What is the relationship between $\beta_1$ and $\gamma_1$? Explain in detail. (8%)

3. A finance department is experimenting with an online pre-course for incoming Master students, to be taken during the summer before the students enroll. Twenty students are randomly assigned to take the pre-course while 16 are assigned to a control group. All of the students take a qualifying exam at the end of their first year. The department would like to test the null hypothesis that the pre-course has no effect against the alternative that the pre-course increases the mean qualifying-exam score. The average scores of students who take the pre-course and of students who do not are

   $\bar{X} = 68.8$ (for students who take the pre-course), and

   $\bar{Y} = 67.8$ (for students who do not take the pre-course).

注意：背面有試題

所別： 財務金融學系 碩士班 甲組(一般生)　　第2頁 / 共4頁

科目： 統計

＊本科考試禁用計算器

The sample variances of scores of students who take the pre-course and of students who do not are $S_X = 400$ and $S_Y = 256$.

(a) What assumptions must hold for the department to be able to use these estimates as the basis for a hypothesis test? (3%)

(b) Can the department reject the null hypothesis that the pre-course has no effect in favor of the alternative that the pre-course increases the mean score of the qualifying exam at a 5% significance level? (Assume that the assumptions you answer in (a) hold.) (7%)

4. Suppose we want to model the number of rainy days in a small town during a 120-day period using a Bernoulli trials process $\{X_i\}_{i=1}^{120}$ with unknown parameter $p$, the probability of a rainy day, where $X_i = 1$ if it rains on day $i$, and $X_i = 0$ otherwise.

(a) What is the expected value of the sample mean $\bar{X}$? (5%)

(b) Is the percentage of rainy days over the 120-day period an unbiased estimator of $p$? Explain. (3%)

(c) What is the variance of the sample mean $\bar{X}$? (4%)

5. A researcher estimates the following two regression models

Model A: $y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \varepsilon_t$,

Model B: $y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t$

where $\varepsilon_t$ and $u_t$ are i.i.d. error terms and $x_{4t}$ is an irrelevant variable which is not related to $y_t$. Will both values of $R^2$ and adjusted $R^2$ be higher for Model B than Model A? Explain your answer. (5%)

6. Let's assume stock return $S_T = S_0 e^{u+\epsilon}$, where $S_0$ and $S_T$ are stock prices are today (time zero) and future time $T$, respectively.

The equality implies $ln(\frac{S_T}{S_0}) = u + \epsilon$. Assuming $\epsilon$ follows a uniform distribution such at $\epsilon \sim U(c, d)$.

(a) Please derive the formula for the expected stock price $E(S_T)$ (5%)

(b) Please derive the formula for the variance of stock price $Var(S_T)$.(5%)

Now, if $\epsilon$ follows a normal distribution. $\epsilon \sim N(0, \sigma^2)$.

(c) Please derive the formula for the expected stock price $E(S_T)$ (5%)

注意：背面有試題

(d) Please derive the formula for the variance of stock price $Var(S_T)$.(5%)

7. Assume a COVID-19 test has been developed to detect COVID-19. Given a person contaminated by COVID-19, the probability of a positive test result is 0.9. For a healthy person, the probability of a negative test result is 0.9. [A positive test indicates this person is contaminated by Covid-19]

   (a) In an area with 50% of the population contaminated by COVID-19, if a positive test result has been observed, what is the probability that this person is contaminated? (4%)

   (b) In an area with 1% of the population contaminated by COVID-19, if a positive test result has been observed, what is the probability that this person is contaminated? (4%)

   (c) Given this example, will you suggest your friend have a flue test when flue is not prevailing? (2%)

8. Let's assume the TWSE (The Taiwan Stock Exchange) index is now 16000 (and the risk-free rate is now zero), and you use its historical data to calculate its distribution. You find the mean of daily return is 0, and the standard deviation is 0.01. (i.e., 1%). Now, you are designing a contract for your client, and the contract is a one-day contract. That is, based on this contract, your client pays the price to buy the contract and gets the indicated payoff tomorrow. (The payoff is listed below.) Given that you fully believe in statistical estimations and believe that historical probability will repeat. Please based only on historical statistical probability, price the following daily contracts or calculate the value of the contract today.

   (a) Please price a contract that will give your client $1 dollar tomorrow if the TWSE index is greater than 16160. (5%)

   (b) Please price a contract that will give your client $1 dollar tomorrow if the TWSE index is greater than 15840. (5%)

   [When we mention "price a contract," here it refers to setting an dollar amount that your client needs to pay today.]

# 國立中央大學113學年度碩士班考試入學試題



所別： 財務金融學系 碩士班 甲組(一般生)

科目： 統計

＊本科考試禁用計算器

9. Consider the population described by the probability distribution of a random variable x below.

| x | | 0 | 2 | 4 |
|---|---|---|---|---|
| Probability of x | | 1/3 | 1/3 | 1/3 |

(a) Find the sampling distribution of the sample mean for a random sample of n = 3 measurements from this distribution. (3%). Find the sampling distribution of the sample median for a random sample of n = 3 observations from this population. (3%)

(b) Show that both the mean and the median are unbiased estimators of μ for this population. (4 %)

注意：背面有試題