國立中央大學114學年度碩士班考試入學試題

系所： 工業管理研究所 碩士班 工業管理組(一般生)　　

科目： 統計學

＊本科考試可使用計算器，廠牌、功能不拘

問答與計算題。無過程者不予計分。

1. Consider three independent U(0,1) random variable V_1, V_2, and V_3. Let $X = V_1^{1/n}$, $Y = V_1^{1/n}V_2^{1/(n-1)}$, $Z = V_1^{1/n}V_2^{1/(n-1)}V_3^{1/(n-2)}$.
   (a) What is the Jacobian of transformation from V_1, V_2, V_3 to X, Y, Z? (10pts)
   (b) What is the joint pdf of X, Y, Z? (10pts)
2. Consider n observations sampled independently from U(0,1), $u_1, u_2, \ldots, u_n$. The order statistics of these n observations are denoted by $u_{(1)}, u_{(2)}, \ldots, u_{(n)}$, where $u_{(1)} < u_{(2)} < \ldots < u_{(n)}$.
   (a) What is the joint pdf of $u_{(1)}, u_{(2)}, \ldots, u_{(n)}$? (5pts)
   (b) What is the pdf of $u_{(n)}$? (5pts)
   (c) What is the joint pdf of $u_{(n)}, u_{(n-1)}$? (10 pts)
   (d) What is the joint pdf of $u_{(n)}, u_{(n-1)}, \ldots, u_{(n-k+1)}$? (10pts)
3. A movie was rated by the viewers who had watched it. The scores range from a low of 1 (very uninteresting) to a high of 5 (very fascinating). A sample of 16 viewers was chosen and their ratings are listed below:

   3　5　4　3　4　5　5　4　3　2　5　4　3　2　3　5

   If the score was rated 4 or 5, the movie was considered Grade "A" for that viewer. The movie production company made the claim that at least 60% of the total viewers would grade the movie as "A". This claim is tested by the movie review company. Use the sample in this problem to conduct the test.
   (a) State the appropriate null and alternative hypotheses for the movie review company. (3pts)
   (b) Compute the value of the test-statistic. (6pts)
   (c) Using $\alpha = 0.05$, would the movie review company conclude that at least 60% of the total viewers would grade the movie as "A"? Explain. (6pts)
4. In a study of innovation in the insurance industry, an economist wished to relate the speed with which a particular insurance innovation is adopted (Y) to the size of the insurance firm (X_1) and the country of firm registered (X_2). The second predictor is qualitative and is composed of 4 classes/countries. The observed data and scatter plot are shown in the following pages.
   (a) Please build a simple linear regression model with the 10 observations of Canada to relate Y to X_1. Show the resulting estimated regression equation. (5 pts)
   (b) For (a), please summarize the testing result in an ANOVA-like table for the significance of the first predictor (X_1) with $\alpha = 0.05$. Please also show the corresponding r^2. (10 pts)
   (c) Please build a simple linear regression model with all the 40 observations regardless of countries to relate Y to X_1. Show the resulting estimated regression equation, the corresponding r^2, and the testing result for the significance of X_1. Explain why r^2 is smaller compared to the one in (b). (15 pts)

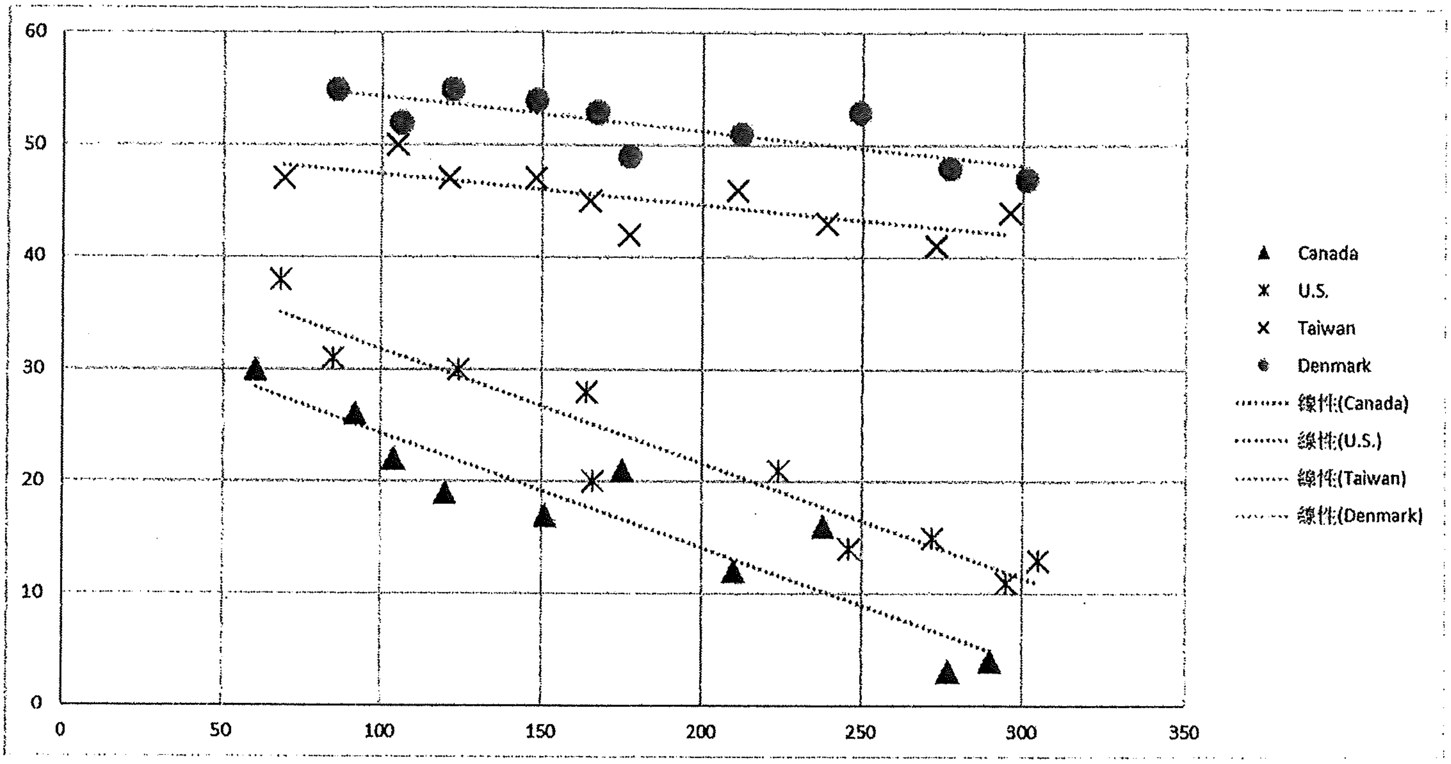
注意：背面有試題

(d) Please build 4 simple linear regression models for 4 countries, and also build a general linear regression model by introducing appropriate dummy variables. (5 pts) (no need to compute the estimated regression equation)

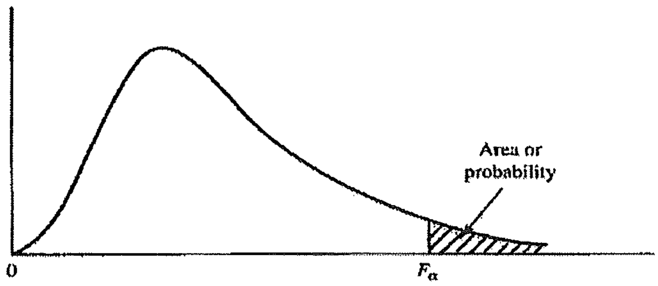
Country	X_1	Y	X_1^2	Y^2	X_1Y	Country	X_1	Y	X_1^2	Y^2	X_1Y
Canada	151	17	22801	289	2567	U.S.	164	28	26896	784	4592
Canada	92	26	8464	676	2392	U.S.	272	15	73984	225	4080
Canada	175	21	30625	441	3675	U.S.	295	11	87025	121	3245
Canada	60	30	3600	900	1800	U.S.	68	38	4624	1444	2584
Canada	104	22	10816	484	2288	U.S.	85	31	7225	961	2635
Canada	277	3	76729	9	831	U.S.	224	21	50176	441	4704
Canada	210	12	44100	144	2520	U.S.	166	20	27556	400	3320
Canada	120	19	14400	361	2280	U.S.	305	13	93025	169	3965
Canada	290	4	84100	16	1160	U.S.	124	30	15376	900	3720
Canada	238	16	56644	256	3808	U.S.	246	14	60516	196	3444
Sum	1717	170	352279	3576	23321	Sum	1949	221	446403	5641	36289

Country	X_1	Y	X_1^2	Y^2	X_1Y	Country	X_1	Y	X_1^2	Y^2	X_1Y
Taiwan	105	50	11025	2500	5250	Denmark	86	55	7396	3025	4730
Taiwan	148	47	21904	2209	6956	Denmark	148	54	21904	2916	7992
Taiwan	211	46	44521	2116	9706	Denmark	167	53	27889	2809	8851
Taiwan	121	47	14641	2209	5687	Denmark	301	47	90601	2209	14147
Taiwan	177	42	31329	1764	7434	Denmark	177	49	31329	2401	8673
Taiwan	239	43	57121	1849	10277	Denmark	277	48	76729	2304	13296
Taiwan	165	45	27225	2025	7425	Denmark	106	52	11236	2704	5512
Taiwan	273	41	74529	1681	11193	Denmark	249	53	62001	2809	13197
Taiwan	296	44	87616	1936	13024	Denmark	212	51	44944	2601	10812
Taiwan	69	47	4761	2209	3243	Denmark	122	55	14884	3025	6710
Sum	1804	452	374672	20498	80195	Sum	1845	517	388913	26803	93920

＊本科考試可使用計算器，廠牌、功能不拘



F TABLE - 1



Entries in the table give F_α values, where α is the area or probability in the upper tail of the F distribution. For example, with 12 numerator degress of freedom, 15 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 2.48$.

Table of $F_{.05}$ Values

Denominator Degrees of Freedom	Numerator Degrees of Freedom																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00