國立中央大學114學年度碩士班考試入學試題

系所：化學工程與材料工程學系 碩士班 不分組(一般生)

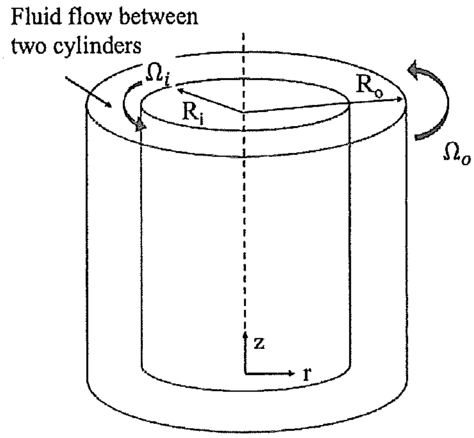
科目：輸送現象與單元操作

＊本科考試可使用計算器，廠牌、功能不拘

計算題（應詳列計算過程，無計算過程者不予計分）

1. **(16 pts) Fluid flows between two cylinders**

An incompressible and Newtonian fluid (density ρ and viscosity μ) is contained between two concentric cylinders of radii R_i (inner) and R_o (outer). A schematic representation of the system is shown in the figure on the right. **Use the coordinates defined in the figure to answer the following questions.**



Assume $v_r = v_z = 0$

(a) **(5 pts)** The two cylinders are fixed initially, and the fluid is correspondingly stationary at $t \leq 0$. When $t > 0$, the two cylinders start to rotate. The inner cylinder rotates at an angular velocity of Ω_i in the positive θ direction, and the outer cylinder rotates at an angular velocity of Ω_o in the positive θ direction. Assume a laminar flow. Please write down the governing equation (in a simplified form) using Navier-Stokes equation and the necessary boundary conditions for solving the velocity profile, v_θ. Assume v_θ is independent of θ and z.

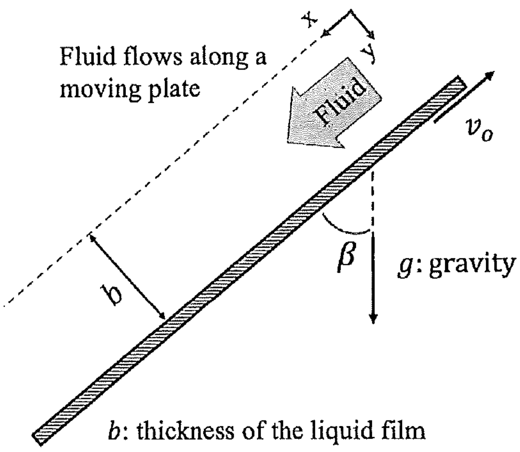
(b) **(5 pts)** Follow **problem 1(a)**, derive the velocity profile v_θ when the fluid reaches to steady-state.

(c) **(2 pts)** Assume the inner and outer cylinders have the same angular velocity: $\Omega_i = \Omega_o = \omega$. Express the velocity profile (v_θ) in terms of ω and r.

(d) **(4 pts)** Sketch velocity (v_θ) and stress profiles ($\tau_{r\theta}$) of the case in problem 1(c) **from the top view.**

2. **(14 pts) Liquid film on an inclined plate**

A Newtonian fluid flows with a film thickness of b on an inclined plate as schematically shown on the right. The plate is moving toward $-x$ direction with speed of v_o, $v_o > 0$. The system has the gravity force pointing down. Assume the flow is a laminar and steady-state flow. **Use the coordinates defined in the figure to answer the following questions.**



(a) **(8 pts)** Derive the velocity profile (v_x) and shear stress profile (τ_{yx}) of the liquid film.

(b) **(2 pts)** Sketch the velocity (v_x) profile if the velocity contribution from v_o and gravity is comparable.

(c) **(2 pts)** Sketch the velocity (v_x) profile if the velocity contribution from v_o is significantly larger than the gravitational force so that $v_x < 0$ for $0 < y < b$.

注意:背面有試題

(d) **(2 pts)** If the plate is moving toward $+x$ direction with speed of v_O, sketch the velocity (v_x) profile if the velocity contribution from v_o and gravity is comparable.

Detailed derivations are needed for all answers.

3. **(40 pts)** An object of irregular shape 1 m long maintained at a constant temperature of 100°C is suspended in an airstream having a free stream temperature of 0°C, a pressure of 1 atm, and a velocity of 120 m/s. The air temperature measured at a point near the object in the airstream is 80°C. A second object having the same shape is 2m long and is suspended in an airstream in the same manner. The air free stream velocity is 60 m/s. Both the air and the object are at 50°C, and the total pressure is 1 atm. A plastic coating on the surface of the object is being dried by this process. The molecular weight of the vapor is 82, and the saturation pressure at 50°C for the plastic material is 0.0323 atm. The mass diffusivity for the vapor in air at 50°C is 2.60×10^{-5} m^2/s. **(a) (8 pts)** Derive the heat diffusion equation based on the energy balance and Fourier's law, with appropriate assumptions. **(b) (8 pts)** Derive the heat convection equation based on the energy balance, with appropriate assumptions. **(c) (6 pts)** Use the equations you derived in (a) and (b) to construct the boundary layer equation in the dimensionless form for the thermal boundary layers, with appropriate approximations and definition of dimensionless groups. **(d) (4 pts)** Based on (c), analogously derive the boundary layer equation for the concentration boundary layer and the corresponding dimensionless groups. **(e) (7 pts)** For the second object, at a location corresponding to the point of measurement on the first object, determine the vapor concentration and partial pressure on the basis of (c) and (d). Explain why heat-mass transfer analogy is applicable to the current case. **(f) (7 pts)** If the average heat flux q'' is 2000 W/m^2 for the first object, determine the average mass flux n''_A ($kg/s{\cdot}m^2$) for the second object.

4. **(30 pts)** An equimolal mixture of A and B with a relative volatility of 2.3 is to be separated into a distillate product with 98.5 percent A, a bottoms product with 2 percent A, and an intermediate liquid product that is 80 percent A and has 40 percent of the A fed. **(a) (12 pts)** Derive the equations for the operating lines in the rectifying, stripping, and middle sections of the column. **(b) (10 pts)** Calculate the amounts of each product per 100 moles of feed, and determine the minimum reflux rate if the feed is liquid at the boiling point. **(c) (8 pts)** How much greater is the minimum reflux rate because of the withdrawal of the side-stream product?

T (K)	ρ (kg/m^3)	c_p ($kJ/kg \cdot K$)	$\mu \cdot 10^7$ ($N \cdot s/m^2$)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ ($W/m \cdot K$)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683

注意:背面有試題

A1. Newton's law of viscosity

Cylindrical coordinates (r, θ, z)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathrm{v})$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathrm{v})$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathrm{v})$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]$$

$$(\nabla \cdot \mathrm{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

A2. The equation of continuity

Cylindrical coordinates (r, θ, z)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical coordinates (r, θ, ϕ)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r}\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

A3. The equation of motion for a Newtonian fluid with constant ρ and μ.

Cylindrical coordinates (r, θ, z)

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$