計算題（應詳列計算過程，無計算過程者不予計分）

1. (10%) The input-output (I/O) relationship of the system is given below, where $x(t)$ represents the input and $y(t)$ represents the corresponding output. For each statement, answer **TRUE** or **FALSE** and **provide justification**.

$$y(t) = \int_0^{\infty} e^{-(\tau+3)} x(t-\tau)d\tau, \ \ t \geq 0$$

(a) (2%) The system is time-varying
(b) (2%) The system is non-causal
(c) (2%) The system is unstable
(d) (4%) Additionally, determine the frequency response of the system's transfer function.

2. (15%) Let $\boldsymbol{S}$ be a linear, time-invariant, and causal (LTIC) system. When the input unit step function $u(t)$, defined for $t \geq 0$, is applied to the system, the corresponding output is $te^{-t}u(t)$. Determine the impulse response function of the system using the time-domain approach and the frequency-domain approach, respectively.

3. (30%) The definition of Fourier Transform (F.T.) in this problem set is

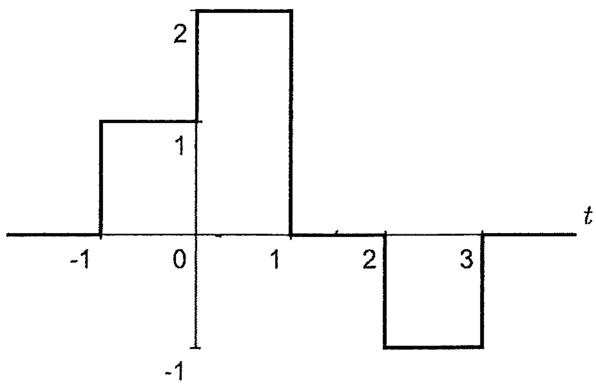
$$F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

and the *sinc* function is defined as

$$sinc(x) = \frac{sin(\pi x)}{\pi x}$$

Answer the following questions:

(a) (10%) Let $x(t)$ be $u(t+1) - u(t-1)$, where $u(t)$ is the unit step function. Obtain the F.T. of $x(t)$. Express your answer in *sinc* function.
(b) (14%) Obtain the F.T. of the following signal $y(t)$.

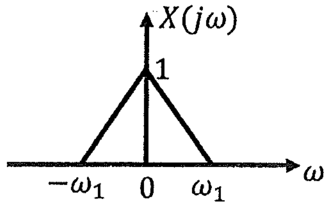


注意：背面有試題

(c) (6%) Following (b), if the spectrum of $y(t)$ is $Y(j\omega)$, obtain the value for

$$\int_{\omega=-\infty}^{\infty} |Y(j\omega)|^2 d\omega.$$

4. (20%) Given a continuous-time signal $x(t)$ with a maximum frequency ω_1 (with spectrum shown here), consider the following sampling schemes for different modulations.



(a) (6%) If $y(t) = x(t)e^{j\omega_1 t}$, what is the **minimum** sampling frequency ω_s required to avoid aliasing in $y(t)$?

(b) (4%) Following (a), what is the **minimum** sampling frequency for $|y(t)|^2$?

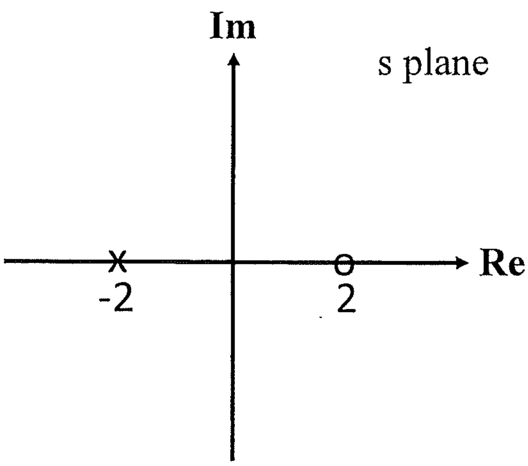
(c) (7%) If $z(t) = x(t)\cos(3\omega_1 t)$, $z(t)$ becomes a band-pass signal. In this case, it is possible to sample at a frequency lower than the Nyquist rate. Without aliasing, what is the **minimum** sampling frequency for $z(t)$?

(d) (3%) Following (c), what type of filter (i.e., low pass, high pass, band pass, or all pass filter) is required to reconstruct $z(t)$?

5. (10%) Sendra and Smith designed an analog linear-time-invariant (LTI) circuit, and they tried to analyze this circuit in a signals-and-systems fashion. They figured out that the transfer function of this LTI circuit owns the following pole-zero plot in the s plane. Note that **x** and **o** represent pole and zero, respectively.

(a) (5%) What type of frequency selective filters (e.g., low pass filter, high pass filter, band pass filter, and all pass filter) is this LTI circuit?

(b) (5%) Find the transfer function of this LTI circuit and its region of convergence (ROC) if it is a causal system.



6. (15%) Via the pole-zero placement method, Oppenheim wants to design a digital LTI system to replace an analog circuit. It is known that this digital system will process signals sampled with a sampling rate of 8000 Hz.

   (a) (5%) To design a digital LTI system that replaces an analog circuit with a maximum magnitude response at the frequency of 1000 Hz, which pole-zero placement below should he use?
   (b) (5%) To design a stable and causal digital LTI system that replaces an analog circuit with a maximum magnitude response at the frequency of 1000 Hz, which pole-zero placement below should he use?
   (c) (5%) To design a digital LTI system that has a pole at infinity, which pole-zero placement below should he use?