計算題　　※計算題需計算過程，無計算過程者不予計分

1. (32%) Let $u(t)$ and $y(t)$ be the input and output, respectively, of a system, which satisfies

$$\frac{dy(t)}{dt} + y(t) = b\int_0^t \cos(\omega(t-\tau))\,u(\tau)d\tau + u(t)$$

where $b, \omega > 0$ are constants.

(a) (6%) Find the transfer function from $u(t)$ to $y(t)$.

(b) (8%) Define $x_1(t) = y(t)$, $x_2(t) = \int_0^t \cos(\omega(t-\tau))\,u(\tau)d\tau$, $x_3(t) = \frac{dx_2(t)}{dt} - u(t)$. Use $x_1(t)$ ; $x_2(t)$ , and $x_3(t)$ as the state variables and find the state-space representation of the system.

(c) (8%) Let $s(t)$ be the unit-step response of the system and suppose that $s(0) = 0$. Find $s(t)$ for $t \geq 0$.

(d) (6%) Let $s(t)$ be defined in part (c). If $s(t)$ is a sinusoidal function with a constant bias, i.e. $s(t) = A\cos(\omega t + \theta) + c$ for $t \geq 0$ and for some constants $A$, $\theta$, and $c$, then what is $b$ in terms of $\omega$?

(e) (4%) Let $h(t)$ be the impulse response of the system. Find $h(0)$.

2. (18%) Consider the feedback control system in Figure 1. Let $G(s) = \frac{1}{s(s+1)}$.

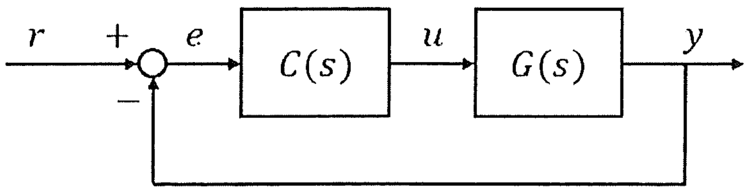



Figure 1: Feedback control system in Problem 2

(a) (6%) Let $C(s) = K > 0$ and $r(t) = 1$ for $t \geq 0$. Find $K$ such that $y(t)$ has the shortest rise time under the condition that the percent maximum overshoot is less than or equal to 10%. Use the formula $t_r \approx \frac{0.8+2.5\xi}{\omega_n}$ to approximate the rise time $t_r$, where $\omega_n$ and $\xi$ are the undamped natural frequency and damping ratio of the closed-loop system, respectively.

(b) (4%) Let $C(s) = K > 0$ and $r(t) = 2t$ for $t \geq 0$. What is the minimum value of $K$ such that the absolute steady-state error $|e_{ss}| = \lim_{t\to\infty}|e(t)|$ is less than or equal to 0.05?

(c) (8%) Let $C(s) = K\frac{s+2}{s^2+8s+20}$, where $K > 0$. Sketch the root locus of the system for $K$ ranging from 0 to $\infty$. Find the range of $K$ such that the closed-loop system is stable.

注意：背面有試題

3. (25%) A unity-feedback system has the open-loop transfer function $L(s)=K\dfrac{s+z}{s(s-1)}$, where $z>0$ and $-\infty<K<\infty$. Also, $L(j\omega)=\dfrac{-K\omega(z+1)+jK(z-\omega^2)}{\omega(\omega^2+1)}$.

Hint: The Laplace transform of $\sin(\omega_0 t)$ is $\dfrac{\omega_0}{s^2+\omega_0^2}$.

(a) (6%) Sketch the Nyquist plot for $K>0$, including asymptotes if available.

(b) (4%) Determine the stability from (a).

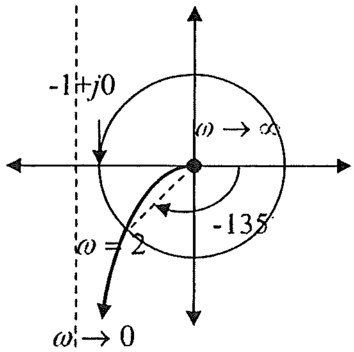
(c) (5%) Sketch the Nyquist plot and the stability for $K<0$.

(d) (2%) Find the steady-state output when the input $=1+\sin(\sqrt{z}t)$ for $t\geq 0$ and $K=1/2$.

(e) (3%) Find the steady-state output when the input $=1+\sin(\sqrt{z}t)$ for $t\geq 0$ and $K=2$.

(f) (5%) Find the steady-state output when the input $=1+e^{-zt}$ for $t\geq 0$ and $K=1$.

4. (25%) A control engineer has the experimental data for a minimum-phase open-loop transfer function in a unity negative feedback system and sketches the corresponding frequency response as the Nyquist plot in the right.




(a) (3%) Sketch the Magnitude-Phase plot of the open-loop transfer function with the data shown on the right figure.

(b) (6%) Determine the open-loop transfer function with minimum order.

(c) (3%) Does the asymptote exist for the Nyquist plot? If yes, find it from your answer in part (b). If not, explain why.

(d) (3%) Determine the closed-loop stability from this plot by General Nyquist criterion.

(e) (10%) Find the Gain Margin, Phase Margin, gain-crossover frequency and phase-crossover frequency and what is the maximum allowable delay time?