單一選擇題，共20題，每題5分。

1. In the vector space R^4, what is the dimension of the subspace spanned by the set $\{(1,0,1,0),(1,2,0,3),(0,-1,-4,1),(2,1,-3,4),(2,3,5,2)\}$?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

2. Let $T: P_5(R) \to R^8$ be linear, where $P_5(R)$ is the vector space consisting of all polynomials with real-valued coefficients and having degree less than or equal to five. If we know that the rank of T is 2 , then what is the nullity of T ?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

3. Let $A, B \in M_{n\times n}(F)$, where $M_{n\times n}(F)$ is the vector space consisting of all $n \times n$ matrices with entries from a field F. Which of the following statements is incorrect?
(A) $\text{rank}(AB) \leq \text{rank}(A)$.
(B) $\text{rank}(AB) \leq \text{rank}(B)$.
(C) $\det(AB) = \det(A) \cdot \det(B)$.
(D) If $\det(AB) \neq 0$, then both A and B are invertible.
(E) If $\det(AB) = 0$, then both A and B are not invertible.

4. Let $A \in M_{n\times n}(F)$ and let A^t be the transpose of A. Which of the following statements is incorrect?
(A) A and A^t have the same determinant.
(B) A and A^t have the same characteristic polynomial.
(C) A and A^t have the same eigenvalues.
(D) A and A^t have the same eigenvectors.
(E) A and A^t have the same diagonalizability, i.e., A is diagonalizable if and only if A^t is diagonalizable.

5. Consider the vector space R^2 endowed with the standard inner product. Let $u = (2,6)$ be a vector in R^2 and let $W = \{(x,y): y = 4x\}$ be a subspace of R^2. Which of the following is the orthogonal projection of the vector u on the subspace W ?
(A) $(24/17, 96/17)$
(B) $(26/17, 104/17)$
(C) $(7/19, 28/19)$
(D) $(11/19, 44/19)$
(E) $(26/23, 104/23)$

6. A vector space is spanned by $\{1, \cos(t), \sin(t)\}$ for $-\pi \le t \le \pi$. If a vector: $v = a \cdot 1 + b \cdot \sin(t) + c \cdot \cos(t)$ is the closest vector in this vector space to a continuous function: $f(t) = t$ for $-\pi \le t \le \pi$, what is this closest vector v? You may need the following integral: $\int t \cdot e^{it} dt = -i(t+i)e^{it} + C$.

(A) $v = 1 + 2\sin(t) - 2\cos(t)$
(B) $v = 1 - 2\cos(t)$
(C) $v = 2\sin(t)$
(D) $v = 1 + 2\sin(t)$
(E) $v = -2\cos(t)$

7. For a 5x5 matrix: $A(t) = \begin{pmatrix} 7 & 1 & -2 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 3 & 4 \\ 2 & 2 & 2\sin t & 2 & 2 \\ 2 & 1 & 0 & 1 & 2 \end{pmatrix}$, which value of t will make both $\det(A(t))$ and $\frac{d}{dt}\det\left(A(t)\right)$ equal 0?

(A) 0
(B) $\pi/6$
(C) $\pi/2$
(D) $3\pi/4$
(E) π

8. A quadratic equation is described as: $x^2 + 8xy + 7y^2 = 225$. Which of the following statement is incorrect?

(A) This quadratic curve is an ellipse.
(B) The curve is centered at the origin.
(C) One of the principal axis is $\frac{1}{\sqrt{5}}(2x - y)$
(D) The other principal axis is $\frac{1}{\sqrt{5}}(x + 2y)$
(E) The shortest distance from this quadratic curve to the origin is 5.

9. Let **T** be a linear operator in $\mathbf{C}^2$ and is defined by $\mathbf{T}(a, b) = (3a + (2 + i)b, (2 - i)a + 7b)$. What kind of operator is T?
   I. Normal,
   II. Self-adjoint,
   III. Unitary,
   IV. Orthogonal.

(A) I only
(B) I and II
(C) I, II, III
(D) I, II, III, IV
(E) None of them

10. For a linear equation system: $\begin{cases} x+2y+z=4 \\ x-y+2z=-11 \\ x+5y=19 \end{cases}$, which of the following statement is incorrect?
(A) The system is consistent.
(B) (-6, 5, 0) is one of the solutions.
(C) The corresponding homogeneous system has more than one solution.
(D) (-10, 2, 6) is a spanning vector to form the subspace generated by the solutions of the corresponding homogeneous system.
(E) (-6, 5, 0) is the minimal solution.

11. Which of the following complex functions is analytic in the complex z-plane, in the open disk defined by $|z| < 1$?
   (A) $1/z$
   (B) $z^{1/2}$
   (C) $\cot(z)$
   (D) e^z
   (E) None of the above

12. Which of the following statements is WRONG about an analytic function $f(z)$ in an open, simply connected domain D? C below refers to a simple path in D going from the complex number 'a' to the complex number 'b'.
   (A) If a = b, the line integral of $f(z)$ along C vanishes.
   (B) The line integral of $f(z)$ along C depends only on 'a' and 'b',
   (C) The function given by $f'(z) + f''(z)$ is analytic in D, too.
   (D) $f'(z)/f(z)$ integrated along C is given by $\ln(f(b)) - \ln(f(a))$.
   (E) $f'(z) + f''(z)$ integrated along C is given by $f(b) + f'(b) - f(a) - f'(a)$.

13. Which of the following power series is NOT an analytic function in the open disk $|z| < 1$, in the complex z-plane?
(A) The geometric series given by $1 + z + z^2 + \ldots.$
(B) The derived series $1 + 2z + 3z^2 + \ldots.$ obtained from the above geometric series.
(C) The integrated series $z + z^2/2 + z^3/3 + \ldots.$ obtained from the above geometric series.
(D) $1 + z + \ldots + z^n / n! + \ldots$
(E) The geometric series given by $1 + 2z + (2z)^2 + \ldots.$

14. Which of the statements is WRONG about sin(i z), where $z = x + i y$ with (x,y) the Cartesian coordinates?
(A) $\sin(i z) = i \sinh(z)$.
(B) $\sin(iz)$ is periodic in x.
(C) $\sin(iz)$ is analytic in the open disk $|z| < 1$.
(D) $\sin(iz)$ is an entire function.
(E) $\sin(iz) = (e^{-z} - e^{z}) /(2i)$.

15. Let f(z) = entire function with a nonvanishing value at $z = 0$, and $g(z) = f(z) / z^2$. Which of the statements is CORRECT about g(z)?
(A) g(z) has a simple pole at $z = 0$.
(B) g(z) has a residue given by f(0) at $z = 0$.
(C) When g(z) is expanded into a Taylor series about $z_0 = 2i$, it has a radius of convergence = 2.
(D) g(z) has a residue given by f'(1) at $z = 1$.
(E) None of the above.

16. Find the solution to $\frac{dy}{dx} + 3y = e^{-x}x^2$. (Note that all c's are constants)

(A). $ce^{-x} + \frac{1}{4}(2x^2 - 2x + 1)e^{-3x}$　(B). $ce^{-x} + \frac{1}{4}(2x^2 + 2x - 1)e^{-3x}$

(C). $ce^{-3x} + \frac{1}{4}(2x^2 + 2x + 1)e^{-x}$　(D). $ce^{-3x} + \frac{1}{4}(2x^2 - 2x + 1)e^{-x}$

(E). $ce^{-3x} + \frac{1}{4}(2x^2 - 2x - 1)e^{-x}$

17. Find the solution to $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+y=\sin x$

(A). $e^{-\frac{x}{2}}(c_1\cos\sqrt{3}x+c_2\sin\sqrt{3}x)-\sin x$

(B). $e^{-\frac{x}{2}}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)-\cos x$

(C). $e^{-\frac{x}{2}}(c_1\cos\sqrt{3}x-c_2\sin\sqrt{3}x)+\sin x$

(D). $e^{-\frac{x}{2}}(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x)+\cos x$

(E). $e^{-\frac{x}{2}}(c_1\cos\frac{\sqrt{3}}{2}x-c_2\sin\frac{\sqrt{3}}{2}x)+\sin x$

18. Find the solution to $x^2\dfrac{d^2y}{dx^2}+y=3x^2$, for $x>0$.

(A). $y=-x^2+\sqrt{x}[c_1\cos(\frac{\sqrt{3}}{2}(\ln x))+c_2\sin(\frac{\sqrt{3}}{2}(\ln x))]$

(B). $y=2x^2-\sqrt{x}[c_1\cos(\frac{\sqrt{3}}{2}(\ln x))+c_2\sin(\frac{\sqrt{3}}{2}(\ln x))]$

(C). $y=x^2-\sqrt{x}[c_1\cos(\frac{\sqrt{3}}{2}(\ln x))+c_2\sin(\frac{\sqrt{3}}{2}(\ln x))]$

(D). $y=-x^2+\sqrt{x}[c_1\cos(\frac{\sqrt{3}}{2}(\ln x))-c_2\sin(\frac{\sqrt{3}}{2}(\ln x))]$

(E). $y=2x^2+\sqrt{x}[c_1\cos(\frac{\sqrt{3}}{2}(\ln x))-c_2\sin(\frac{\sqrt{3}}{2}(\ln x))]$

19. Let $f(t)=t-[t]$, where $[t]$ is the largest integer that is not larger than t. Find the Laplace transform of $f(t)$.

(A). $\dfrac{1-e^{-s}(1+s)}{s^2(1-e^{-s})}$ (B). $\dfrac{1+e^{-s}(1+s)}{s^2(1-e^{-s})}$ (C). $\dfrac{1-e^{-s}(1+s)}{s(1-e^{-s})}$

(D). $\dfrac{1+e^{-s}(1-s)}{s^2(1-e^{-s})}$ (E). $\dfrac{1-e^{-s}(1-s)}{s(1-e^{-s})}$

注意：背面有試題

20. Find the inverse Laplace transform of $\frac{1}{s^3+1}$

(A). $\frac{1}{3}e^{-t}+\frac{1}{3}e^{\frac{t}{2}}\cos(\frac{\sqrt{3}t}{2})+\frac{\sqrt{3}}{3}e^{\frac{t}{2}}\sin(\frac{\sqrt{3}t}{2})$

(B). $\frac{1}{3}e^{-t}-\frac{1}{3}e^{\frac{t}{2}}\cos(\frac{\sqrt{3}t}{2})+\frac{\sqrt{3}}{3}e^{\frac{t}{2}}\sin(\frac{\sqrt{3}t}{2})$

(C). $\frac{1}{3}e^{-t}-\frac{1}{3}e^{\frac{t}{2}}\cos(\frac{\sqrt{3}t}{2})-\frac{\sqrt{3}}{3}e^{\frac{t}{2}}\sin(\frac{\sqrt{3}t}{2})$

(D). $-\frac{1}{3}e^{-t}-\frac{1}{3}e^{\frac{t}{2}}\cos(\frac{\sqrt{3}t}{2})+\frac{\sqrt{3}}{3}e^{\frac{t}{2}}\sin(\frac{\sqrt{3}t}{2})$

(E). $-\frac{1}{3}e^{-t}+\frac{1}{3}e^{\frac{t}{2}}\cos(\frac{\sqrt{3}t}{2})-\frac{\sqrt{3}}{3}e^{\frac{t}{2}}\sin(\frac{\sqrt{3}t}{2})$