

**I. Multiple Choice -單選題 (50 points/25 questions - 每題正確 2 分，錯誤則倒扣 0.5 分)**

- Proof by counterexample is a technique you can use to
  - prove that an algorithm is correct;
  - prove that an algorithm is incorrect;
  - prove that an algorithm has a certain time complexity;
  - all of the above;
  - none of the above.
- Which of the following statement is correct?
  - $f(n) = \Theta(g(n))$  implies  $h(f(n)) = O(h(g(n)))$  so long as  $h(n)$  is a strictly increasing function.
  - $f(n) + g(n) = \Theta(\max\{f(n), g(n)\})$ .
  - $f(n) = o(g(n))$  and  $g(n) = o(f(n))$  implies  $f(n) = \Theta(g(n))$ .
  - all of the above.
  - none of the above.
- Suppose algorithms A1 and A2 are both  $\Theta(n^2)$ . If both are coded in the same programming language, compiled with the same compiler, and the executables run on the same computer with the same input, will both execute at the same speed?
  - definitely yes
  - definitely no
  - mostly yes
  - mostly no
  - not necessarily
- For the following recurrence relation, what would be the asymptotic bound of the function  $T(n)$ ?
 
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n/5) + O(n^2) & \text{otherwise} \end{cases}$$
  - $T(n) = \Theta(n^2)$
  - $T(n) = \Theta(n^{\log_2 5})$
  - $T(n) = \Theta(n^{\log_5 2})$
  - $T(n) = \Theta(n^{\log_2 5} \log n)$
  - $T(n) = \Theta(n^2 \log n)$
- The numbers 5, 10, 3, 6, 2, and 4 are inserted sequentially into an empty binary search tree. What is the height of the tree?

注：背面有試題  
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所別：資訊工程學系碩士班 不分組(一般生) 科目：資料結構與演算法 共 7 頁 第 2 頁  
資訊工程學系軟體工程碩士班 不分組(一般生)

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答

- A. 1
  - B. 2
  - C. 3
  - D. 4
  - E. 5
6. Which of the following input is the worst-case scenario (i.e., requires the most number of comparisons) for the merge sort?
- A. The input numbers are in ascending order.
  - B. The input numbers are in descending order.
  - C. The input numbers are almost sorted – only few are not in the right position.
  - D. The input numbers are in random order.
  - E. The order of the input does not matter.
7. Which of the following traverse results can uniquely determine a binary tree?
- A. postorder and inorder traverses
  - B. preorder and postorder traverses
  - C. depth-first and preorder traverses
  - D. breadth-first and inorder traverses
  - E. depth-first and inorder traverses
8. Which of the sorting algorithm is best suited for the updates (i.e., insertions as well as modifications) of only a few entries in a database containing a large number of entries sorted by a particular key.
- A. Insertion sort
  - B. Bubble sort
  - C. Quicksort
  - D. Mergesort
  - E. Heapsort
9. If column major is adopted in a two-dimensional array with 10 rows and 10 columns and  $\alpha$  indicates the memory address of  $A[0][0]$ , which of the following is the memory address of the element  $A[3][8]$ ?
- A.  $\alpha+38$
  - B.  $\alpha+83$
  - C.  $\alpha+72$
  - D.  $\alpha+27$
  - E. none of the above
10. When an array of size  $n$  is used to implement a circular queue and we would like to utilize all the  $n$  spaces in the array, which of the following may be an illegal operation?
- A. an enqueue when  $\text{front} = \text{rear}$
  - B. a dequeue when  $\text{front} = \text{rear}$
  - C. an enqueue when  $\text{rear} = 0$
  - D. all of the above
  - E. none of the above

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11. Which of the following postfix expression is equivalent to the infix expression “(A + B) \* C - D”?  
A. AB+\*CD-  
B. AB\*+CD-  
C. CD-AB+\*  
D. CD-\*AB+  
E. AB+C\*D-
12. One possible way to enforce automatic garbage collection (revocation of objects no longer used) in Java is to use reference count for each object. Which of the following is the issue of this approach?  
A. storage overhead  
B. computation overhead  
C. the reference count of some objects may never reach zero  
D. all of the above  
E. none of the above
13. Priority queues can be used for sorting or task scheduling. When it comes to the implementation, priority queues are normally done by arrays because:  
A. the index of parents and children of a particular node can be identified easily;  
B. the dequeue and enqueue operations can be accomplished in  $O(1)$ ;  
C. the memory can be better utilized;  
D. the sequential search for a particular value is faster;  
E. none of the above.
14. Radix sort is an efficient sorting algorithm for integers. Which of the following statements about radix sort is incorrect?  
A. In each pass of the radix sort, keys are placed into queues based on the value of a specific digit.  
B. The passes in radix sort start from the least significant digit to the most significant digit.  
C. The time complexity of radix sort is  $O(nk)$ , where  $n$  represents the number of keys to be sorted and  $k$  is the average key length.  
D. Radix sort is asymptotically faster than bucket sort.  
E. Radix sort is different from heap sort and merge sort in the sense that it does not rely on the comparisons of keys.
15. Which of the following statements about graph is incorrect?  
A. A cycle in a graph must be a simple path.  
B. A graph is strongly connected if and only if each pair of nodes have more than one path between them.  
C. The strongly connected components of a graph can be identified by depth-first search.  
D. The sum of degree of nodes in a graph equals to twice of the number of edges.  
E. The adjacent matrix is more suitable for graph with more edges than the adjacency lists.

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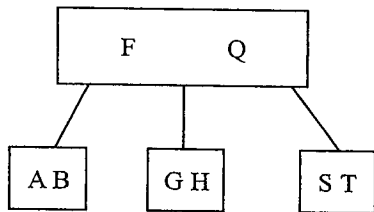
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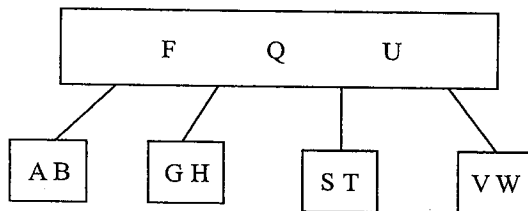
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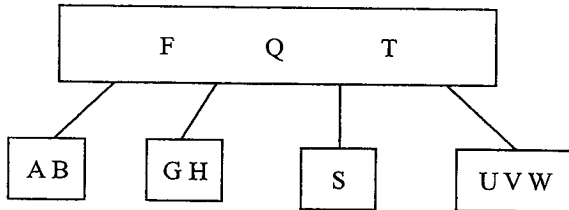
16. Binary search trees cannot guarantee worst-case performance, as it may be skewed. To remedy this, a number of balanced search tree data structures have been proposed. Which of the following is not one of them.
- A. AVL trees
  - B. Red-Black trees
  - C. B trees
  - D. B+ trees
  - E. none of the above
17. 11. Which of the following is a legal max-heap?
- A. {67, 45, 19, 22, 43, 16}
  - B. {67, 43, 19, 22, 45, 16}
  - C. {14, 18, 27, 19, 63, 48}
  - D. {45, 22, 67, 8, 34, 52}
  - E. {99, 42, 7, 16, 4, 48}
18. Starting with the following B-tree ( $t=2$ ), which of the following happens when you insert U,V,W? (10 points)



A.



B.



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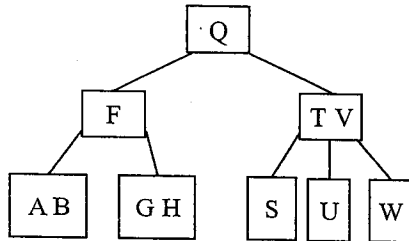
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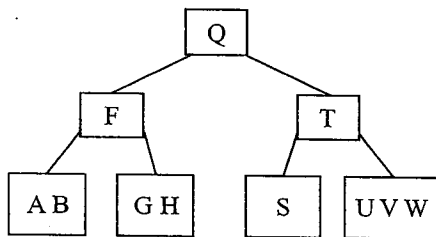
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C.



D.



E. none of the above

19. Suppose there is a text file containing text composed only by the five letters  $\{a, b, c, d, e\}$  with the corresponding probability distribution  $\{0.05, 0.10, 0.12, 0.13, 0.60\}$ . Which of the following encoding will produce the shortest encoding of the original text?
- A.  $a = 000, b = 001, c = 010, d = 011, e = 100$ ;  
 B.  $a = 000, b = 001, c = 010, d = 011, e = 1$ ;  
 C.  $a = 0, b = 100, c = 110, d = 101, e = 111$   
 D.  $a = 100, b = 010, c = 10, d = 00, e = 11$   
 E.  $a = 111, b = 110, c = 1, d = 101, e = 100$ .
20. When multiplying matrix  $A_1A_2A_3$ , where  $A_1$  is of size  $10 \times 100$ ,  $A_2$  is  $100 \times 5$ , and  $A_3$  is  $5 \times 50$ , the minimum number of multiplications needed would be:
- A. 75000;  
 B. 7500;  
 C. 250000;  
 D. 12500;  
 E. none of the above.
21. If hash function  $h(k) = k \bmod 13$  is used along with linear probing, and the numbers  $\{15, 33, 3, 72, 28\}$  are inserted into the array of size 13, which of the following statement is incorrect?
- A. The content of the array will be  $[\text{nil}, \text{nil}, 15, 3, 28, \text{nil}, \text{nil}, 33, 72, \text{nil}, \text{nil}, \text{nil}, \text{nil}]$ .  
 B. There are 2 collisions in the insertion process.  
 C. The linear probing suffers from primary cluster issue.  
 D. When search for a particular key, the search process cannot terminate until it finds an empty space behind the index  $h(\text{key})$  in the array.  
 E. The cluster issue can be completely resolved if quadratic probing is adopted for collision resolution.

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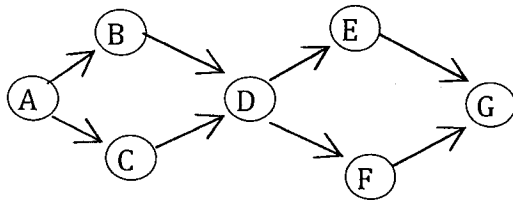
\*請在試卷答案卷(卡)內作答

22. The disjoint sets can be implemented by array. Assume that Union by size and Find with path compression are used, given elements 1, 2, 3, 4, 5 and 6, which of the following could be the content of the array after the following operations.
- Union(1, 2);  
Union(3, 4)  
Union(1, 3)  
Find(4);
- A. [-4, 1, 1, 1, -1, -1];  
B. [0, 1, 1, 1, 0, 0]  
C. [2, -4, 1, 1, -1, -1]  
D. [2, 3, 4, 3, 0, 0]  
E. none of the above
23. Which of the following algorithms is most efficient in finding the shortest path from one node to another in an arbitrary weighted graph with no negative cycle?
- A. Dijkstra's algorithm  
B. Dijkstra's algorithm with Fibonacci heaps  
C. Bellman-Ford algorithm  
D. Floyd-Warshall algorithm  
E. none of the above
24. Both of the following programs (pseudocode) compute the Fibonacci number. Which of the following statements is correct.
- ```
int Fib1(int i){
    if i = 0 or i = 1
        return 1;
    else
        return Fib1(n-1) + Fib1(n-2);
}
int Fib2(int n){
    if i = 0 or i = 1
        return 1;
    fib0 = fib1 = 1;
    for (k = 2, k <= n; k++){
        int temp = fib0 + fib1;
        fib0 = fib1;
        fib1 = temp;
    }
    return fib1;
}
```
- A. The first program runs faster because it is short.  
B. The first program runs faster because it is recursive.  
C. The time complexity of the first program is exponential.  
D. In general recursive programs are easier to optimize compared with the iterative ones.

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E. The recursive programs are easier to debug.

25. How many topological orders does the following graph have?



- A. 2
- B. 4
- C. 6
- D. 8
- E. none of the above

II. In the following problems, if you are asked to design an algorithm, besides words description, you should also use pseudocode (not program) to present your algorithm in a clear and succinct way.

1. Given two arrays  $x[1] \dots x[m]$  and  $y[1] \dots y[n]$ , design an algorithm to find two elements  $x[i]$  and  $y[j]$  such that the absolute value  $|x[i] - y[j]|$  is minimized over all possible pairs of  $x$  and  $y$  elements. A brute force algorithm needs  $O(n^2)$  time. Your algorithm should be far more efficient than that. Analyze the time complexity of your algorithm. (10 points)
2. Given an array of  $n$  elements and a key value  $k$ , design an algorithm to rearrange the array so that all the elements with keys equal to  $k$  are put together and in place with smaller elements to the left and larger elements to the right. Your algorithm should run in  $O(n)$  time and use only  $O(1)$  auxiliary memory space. (15 points) (Hint: Recall the technique used to partition elements in quicksort.)
3. Let  $G=(V, E)$  be a connected, undirected graph. For each edge  $(u, v) \in E$ , there is a weight  $w(u, v)$  specifying the cost to connect vertices  $u$  and  $v$ . The **minimum-spanning-tree (MST) problem** is to find a spanning tree whose total edge weight is minimized.
  - a) Describe any algorithm you know for solving the MST problem. Analyze the algorithm and explain why it works. (10 points)
  - b) Given a graph  $G$  and a minimum spanning tree  $T$  of  $G$ , suppose that we decrease the weight of one of the edges not in  $T$ . Give an  $O(|V|)$  time algorithm for finding the minimum spanning tree in the modified graph. (7 points)
  - c) Suppose that a graph  $G$  has a minimum spanning tree already computed. How quickly can we update the minimum spanning tree if we add a new vertex and incident edges to  $G$ ? (8 points)