所別：電機工程學系碩士班 電波組(一般生)　科目：工程數學(不含複變)　共 / 頁 第 / 頁

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

1. (20%) Solve the following differential equation:

   $y'(\sinh 3y - 2xy) = y^2$

2. (20%) Calculation of Convolution by integrating the following:
   $t * e^{at}$

3. (20%) Find the orthogonal projection of $\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ onto Span$\{\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\}$.

4. (20%) Compute $\det(B^2)$, where $B = A^{-1}$ and $A = \begin{bmatrix} 0 & 2 & 0 & -2 \\ 1 & 5 & 3 & -5 \\ 2 & -7 & 6 & 4 \\ -1 & 3 & 2 & -2 \end{bmatrix}$.

5. For two continuous-time signals $x(t)$ and $h(t)$, the $x(t)$ is an impulse train with inter-impulse interval T, and the $h(t)$ is a rectangular wave with unity gain from $t = 0$ to $t = T_0$ ($T_0 < T$). The mathematical representations of $x(t)$ and $h(t)$ are $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$, where k belongs to integer, and $h(t) = u(t) - u(t - T_0)$, where $u(t)$ is a unit step function with $u(t) = 1$ for $t \geqq 0$. The convolution output $y(t)$ of these two signals is $y(t) = x(t) * h(t)$, in which the $*$ represents the continuous-time convolution operator. Please answer the following questions:

   (a) (5%) Is $y(t)$ a periodic signal or non-periodic signal?

   (b) (15%) Please determine the coefficients of Fourier series for $y(t)$, if $y(t)$ is a periodic signal. Otherwise, please determine the complex Fourier transform for $y(t)$, if $y(t)$ is a non-periodic signal.