一.填充題.共60分.（只需按標碼甲,乙,丙…等填出答案即可）

1. （7分）

    Let $f(x)$ be a differentiable function on $\mathbb{R}$ satisfying

    $$f(x^2) = 1 + \int_0^{x^2} f(y)(1 - \tan y)dy$$

    for all $x \in \mathbb{R}$. Then $f(\pi) = $ __甲__ .

2. （7分）

    Let L be the line tangent to the polar curve $r(\theta) = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$ at $\theta = 0$. The equation of L in x and y is __乙__ .

3. （7分）

    Evaluate the improper integral $\int_0^\infty \frac{dx}{(x+1)[x^2+(x+1)^2]}$ by transforming it into a definite integral of the form $\int_0^1 \frac{ay+b}{\alpha y^2+\beta y+\gamma}dy$ via an appropriate 1-1 onto differentiable function $[0,\infty) \xrightarrow{y=f(x)} [0,1)$. Answer: __丙__ .

4. （7分）

    Evaluate $\int_0^{\frac{\pi}{3}} \frac{1}{\sin x - \cos x - 1}dx = $ __丁__ .

5. （8分）

    Let $p(x) = x^6 + 2x^5 - x + 1$. Find $\lim_{x\to\infty}\{(p(x))^{1/6} - x\} = $ __戊__ .

6. （8分）

    Evaluate $\int\int_\Omega xy\,dxdy$, where Ω is the region in the first quadrant bounded by the curves: $x^2 + y^2 = 4, x^2 + y^2 = 9, x^2 - y^2 = 1, x^2 - y^2 = 4$. Answer: __己__ .

7. （8分）

    Evaluate the line integral $\int_C (x^2 + 6xy - 2y^2)dx + (3x^2 - 4xy + 2y)dy$ along the path $C : y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$. Answer: __庚__ .

8. （8分）

    Find the volume of the solid T bounded above by the plane $z = 2y$ and below by the paraboloid $z = x^2 + y^2$. Answer: __辛__ .

二·計算証明題·共40分（需寫出計算及証明過程，否則不予計分）

1. （10分）

   Find

   $$\lim_{n \to \infty} \left(\sum_{k=1}^{n} \frac{n}{k^2 + n^2} \right).$$

2. （10分）

   Let $0.a_1 a_2 a_3 a_4 \cdots$ be the decimal expansion of the rational number $\frac{5}{7}$. Let $b_k = a_{2^k}, k = 1, 2, \cdots$. The decimal $0.b_1 b_2 b_3 b_4 \cdots$ also represents a rational number $\frac{a}{b}$. Find $\frac{a}{b}$.

3. （10分）

   Find the shortest distance from the point $(1, 2, 0)$ to the elliptic cone $z = \sqrt{x^2 + 2y^2}$.

4. （10分）

   Evaluate the surface integral $\int \int_S (x^4 + y^4 + z^4) d\sigma$, where $d\sigma$ is the surface element and $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$.