# 國立中央大學102學年度碩士班考試入學試題卷

所別：大氣科學學系大氣物理碩士班 不分組(一般生)　科目：應用數學　共 1 頁 第 1 頁
大氣科學學系大氣物理碩士班 不分組(在職生)

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

參考用

1. Let $\vec{A}=5\vec{i}-3\vec{j}+\vec{k}$, $\vec{B}=6\vec{i}+4\vec{j}-2\vec{k}$, find (a) $\vec{A}+\vec{B}$; (b) The angle between $\vec{A}$ and $\vec{B}$; and (c) The size of the parallelogram (平行四邊形) formed by $\vec{A}$ and $\vec{B}$. (9%)

2. $V$ is the volume inside a sphere with radius of 2, compute the surface integration of $\vec{F}=(x\vec{i}+y\vec{j}+z\vec{k})$ over the surface of this sphere. (6%)

3. The so-called stream function $\psi$ can be used to define the wind components $u$ and $v$ by $u=-\partial\psi/\partial y$ and $v=\partial\psi/\partial x$. Explain why the wind fields defined this way represent the non-divergent part of the velocity field. Please also use $\psi$ to represent the vertical vorticity. (10%)

4. Find an eigenbasis and diagonalize.

$$\begin{bmatrix} -12 & 22 & 6 \\ 8 & 2 & 6 \\ -8 & 20 & 16 \end{bmatrix}$$ (10%)

5. Solve the following problems.

a. $y'+xy=xy^{-1}$, $y(0)=3$;

b. $y''+4y'+5y=e^{-t}\cos t$, $y(0)=0$, $y'(0)=1$;

c. $y(t)+2e^{t}\int_0^t y(\tau)e^{-\tau}d\tau=te^{t}$ ;

d. $y''+8y'+(\lambda+16)y=0$, $y(0)=0$, $y(\pi)=0$

(40%)

6. Find the Fourier transform of the following function

$$f(x)=\begin{cases} e^{x} & -a<x<a \\ 0 & otherwise \end{cases}$$

(10%)

7. Find the potential in the rectangle $0\le x\le 20$, $0\le y\le 40$ whose upper side is kept at potential 110V and whose other sides are grounded.

(15%)