科目：工程數學D(3007)

校系所組：中央大學系統生物與生物資訊研究所
交通大學電子研究所(甲組)
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## Part I: Differential Equations

一、(15%) Let $X(t)$ be a fundamental matrix of $\dot{x}(t) = A(t)x(t)$, where $A(t) \in \Re^{n\times n}$ is continuous everywhere. Please determine whether or not the following statements are true (**MUST WITH REASON OR COUNTER EXAMPLE**).

(一) (5%) Both $X(t)C$ and $CX(t)$ are fundamental matrices provided $C \in \Re^{n\times n}$ is a nonsingular matrix.

(二) (5%) If $Y(t)$ is also a fundamental matrix, then there is a unique nonsingular matrix $C$ such that $Y(t) = X(t)C$.

(三) (5%) The state transition matrix $\Phi(t,t_0) := X(t)X^{-1}(t_0)$ is uniquely determined no matter what fundamental matrix $X(t)$ is chosen.

二、(15%) Consider the initial value problem:

$$x''(t) + p_0x'(t) + q_0x(t) = f(t),\ t \ge 0 \text{ and } x(0) = x'(0) = 0.$$

(一) (5%) Determine $p_0$ and $q_0$ so that the solution $x(t)$ can be expressed as $x(t) = \int_0^t e^{-(t-\tau)}\sin(t-\tau)f(\tau)d\tau,\ t \ge 0$.

(二) (5%) Under the same conditions, please also compute $x(t)$ when $f(t) = e^{-t}\cos(t)$.

(三) (5%) Compute $x(t)$ when $p_0 = 2$, $q_0 = 1$ and $f(t) = e^{-t}$.

三、(10%) Consider the differential equation

$t^2x''(t) - t(t+2)x'(t) + (t+2)x(t) = 0,\ t > 0$. Let $r_1$ and $r_2$ be the two roots of the associated indicial equation with $r_1 \ge r_2$.

注意：背面有試題

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(一) (5%) Find $r_1$ and $r_2$.

(二) (5%) Find a second solution $x_2(t)$ with $x_2(0)=0$ and $x_2'(0)=1$, which is linearly independent of the solution $x_1(t)=t$.

四、(10%) Solve the following differential equations:

(一) (5%) $y(x)y''(x)=(y'(x))^2$.

(二) (5%) $x^3y'''(x)+2x^2y''(x)-6xy'(x)=0$.

## Part II: Linear Algebra

(*Please write ALL of your answer in English*)

五、(25%) Let $V$ be a subspace of $\mathbb{R}^5$ generated by

$$\left\{\begin{bmatrix}1\\3\\-3\\-1\\-4\end{bmatrix},\begin{bmatrix}1\\4\\-1\\-2\\-2\end{bmatrix},\begin{bmatrix}2\\9\\0\\-5\\-2\end{bmatrix}\right\}$$

and $W$ be a subspace generated by

$$\left\{\begin{bmatrix}1\\6\\2\\-2\\3\end{bmatrix},\begin{bmatrix}2\\8\\-1\\-6\\-5\end{bmatrix},\begin{bmatrix}1\\3\\-1\\-5\\-6\end{bmatrix}\right\}.$$

(一) (10%) Find a basis and dimension for V+W. You must justify your answer mathematically.

(二) (15%) Find a basis and dimension for V∩W. You must justify your answer mathematically.



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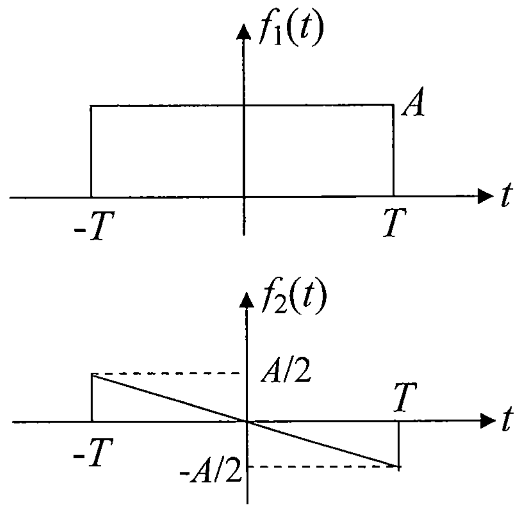
六、(10%) True or False. You must justify your answer mathematically.

(一) (5%) Two matrices that represent the same linear transformation $T$: $V \rightarrow V$ with respect to different bases are not necessarily similar.

(二) (5%) The standard basis for $\mathbb{R}^n$ will always make the coordinate matrix for the linear transformation T the simplest matrix possible.

七、(15%) Assume we have two signals $f_1(t)$ and $f_2(t)$ as shown below




(一) (5%) Is $f_1(t)$ and $f_2(t)$ linearly independent? Please show all work.

(二) (10%) Assume we define

$$\langle f_1(t), f_2(t)\rangle \triangleq \int_0^T f_1(t) f_2(t)\,dt,$$

$$\|f_1(t)\|_2^2 \triangleq \langle f_1(t), f_1(t)\rangle, \text{ and}$$

$$d\left(f_1(t), f_2(t)\right) \triangleq \|f_1(t) - f_2(t)\|_2^2.$$

If $f(t)$ is defined as $f(t) = A\left(1 - \frac{|t|}{T}\right)$, find the coefficients $k_1$ and $k_2$ such that $d\left(k_1 f_1(t) + k_2 f_2(t), f(t)\right)$ has the smallest value.