國立中央大學100學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生)　科目：數理統計　共 一 頁　第 一 頁

本科考試可使用計算器，廠牌、功能不拘

＊請在試卷答案卷（卡）內作答

參考用

1. Let $X_1,\dots,X_n$ be i.i.d. random variables with p.d.f. $f(\cdot;\theta_1,\theta_2)$ given by

$$f(x;\theta_1,\theta_2)=\frac{1}{\theta_2}\exp(-\frac{x-\theta_1}{\theta_2}),$$

where $x>\theta_1, \theta=(\theta_1,\theta_2)'\in\Omega=\Re\times(0,\infty)$.

Find the MLE's of θ_1,θ_2. (15%)

2. Let X_1 and X_2 have the joint p.d.f. $f(x_1,x_2)$ described as follows:

(x_1,x_2)	(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$f(x_1,x_2)$	1/18	3/18	4/18	3/18	6/18	1/18

and $f(x_1,x_2)$ is equal to zero elsewhere.

(a) Find the marginal probability density functions for X_1 and X_2. (10%)

(b) Find the conditional mean of X_1 given $X_2=x_2$. (10%)

3. Let Y_1 and Y_2 be two independent unbiased estimators of θ. The variance of Y_1 is twice variance of Y_2. Find the constants of k_1 and k_2 so that $k_1Y_1+k_2Y_2$ is an unbiased estimator with smallest possible variance for such a linear combination. (15%)

4. Let X and Y be random variables with means μ_1,μ_2; variances σ_1^2,σ_2^2; and correlation coefficient ρ. Show that the correlation coefficient of $W=aX+b$, $a>0$, and $Z=cY+d$, $c>0$, is ρ. (15%)

5. Let X_1 and X_2 be independent random variables distributed as exponential with parameter λ=1. The p.d.f. $f(x)=\exp(-x)$, $x>0$.

(a) Derive the p.d.f. of X_1+X_2 and X_1/X_2, respectively. (10%)

(b) Show that X_1+X_2 and X_1/X_2 are independent. (10%)

6. Let us assume that the life of a tire in miles, say X, is normally distributed with mean θ and standard deviation 5000. Past experience indicates that θ=30000. The manufacturer claims that the tires made by a new process have mean θ> 30000. Let us check his calim by testing

$$H_0:\theta\le 30000 \text{ vs. } H_1:\theta>30000.$$

We shall observe n independent values of X, say $x_1,\dots,x_n$, and we shall reject H_0 if and only if $\bar{x}\ge c$. Determine sample size n and c so that power is 0.98 when θ=35000 at the level of significance α=0.01. (Note that $z_{0.01}=-2.326, z_{0.98}=2.05$) (15%)