國立中央大學100學年度碩士班考試入學試題卷

所別：太空科學研究所碩士班 不分組(一般生)　科目：應用數學　共 2 頁 第 1 頁
本科考試禁用計算器
*請在試卷答案卷（卡）內作答

注意：作答時，請寫出計算步驟或用文字說明如何獲得答案。
如果只列出最後答案，卻沒有文字說明或計算步驟，該題將不予計分。

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1. (15 points) Let functions $v_x(t)$ and $v_y(t)$ satisfy the following ordinary differential equations:

$$m\frac{dv_x(t)}{dt} = qBv_y(t)$$

$$m\frac{dv_y(t)}{dt} = q[E - Bv_x(t)]$$

where m, q, E, and B are positive constants. Please find the solution of the functions $v_x(t)$ and $v_y(t)$ with initial conditions: $v_x(t=0)=0$ and $v_y(t=0)=0$.

2. (10 points) Let function $f(x,v)$ satisfy the following partial differential equation

$$v\frac{\partial f(x,v)}{\partial x} - \frac{d\phi(x)}{dx}\frac{\partial f(x,v)}{\partial v} = 0$$

Let $\phi(x) = \sin(\pi x/2)$.

(a) Find the general solution of $f(x,v)$ in the interval $1 \le x \le 5$.

(b) Let us consider a special case, in which $f(x,v)$ at given x and v are listed in Table 1. Can we determine the value of $f(x,v)$ at $x=3$ and $v=2$ from Table 1? If we can, what is the value of $f(x,v)$ at $x=3$ and $v=2$?

Table 1

x	v	$f(x,v)$
1	1	10
1	0	9
2	1	8
2	0	7
3	1	6
3	0	5

3. (30 points) Evaluate the following integrals

(a) $I_1 = \int_{-\infty}^{\infty} x^2 \exp[-\frac{(x-\mu)^2}{2\sigma^2}]dx$, where $\sigma > 0$ and $\mu > 0$

(b) $I_2 = \int_0^{2\pi} \frac{dx}{\cos x + i5\sin x + 5}$, where $i = \sqrt{-1}$

注意:背面有試題

參考用

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4. (15 points)

Let $M=\begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ c & d & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix}$, where $a>b>0$ and $d>c>0$,

(a) Find the determinate of the matrix M

(b) Find M^{-1} (the inverse matrix of M)

(c) Verify your results by showing that $MM^{-1}=I$, where I is a 4×4 unit matrix.

5. (20 points)

Let $M=\begin{bmatrix} 1 & 0 & -i \\ 0 & 3 & 0 \\ i & 0 & 1 \end{bmatrix}$, where $i=\sqrt{-1}$

(a) Find the eigen values and the corresponding eigen vectors of the matrix M.

(b) The matrix M can be decompose into $M=S\Lambda S^H$, where S^H is the complex conjugate of the transpose of the matrix S, and Λ is a diagonal matrix. Determine the matrices Λ and S, and verify your results by showing that $M=S\Lambda S^H$.

6. (10 points) Let us consider a vector field $\mathbf{V}=\mathbf{e}_x(y^2\operatorname{sech}y)+\mathbf{e}_y(x\tanh x)$, where the boldface font indicates that the variable is a vector. The vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the x and y directions, respectively. Please evaluate the following vector differentiations $\nabla\times(\mathbf{V}\cdot\nabla\mathbf{V})=?$

注意:背面有試題