國立中央大學100學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組(一般生)　科目：微積分　共 1 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷（卡）內作答

(30%) 1. Define a function of y as follows:

$$f(y) = \begin{cases} p\eta_1 e^{-\eta_1 y} & y \geq 0, \\ q\eta_2 e^{\eta_2 y} & y < 0, \end{cases}$$

where $p, q \geq 0$, $p + q = 1$, $\eta_1, \eta_2 > 0$. Please answer the following questions.

(a)Compute the integral: $\int_{-\infty}^{\infty} y f(y) dy$.

(b) Define the cumulative function $F(y) = \int_{-\infty}^{y} f(t) dt$.

(20%) 2. Let's define a function of x as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty,$$

where $\sigma \in R^+$, and then we have the following equation:

$$\int_{-\infty}^{\infty} \mathrm{Max}(e^x - K, 0) f(x) dx = e^{\mu + \frac{\sigma^2}{2}} \Phi(D) - K\Phi(D - \sigma),$$

where $\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$, $-\infty < a < \infty$, and K is a constant. Please compute D.

(15%) 3. Please compute $\frac{\partial C(\sigma)}{\partial \sigma}$, where $C(\sigma)$ is defined as follows:

$$C(\sigma) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$

where $d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, and $\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$.

(15%) 4. Solve the equation: $xy' = x^2 + Cx^3$, $\quad x > 0$, given the initial condition $y(1) = 2$.

(20%) 5. If X has the probability density function $f(x) = \frac{1}{4}$, $-1 < x < 3$, zero elsewhere, find the probability function of $Y = X^2$.

參考用