國立中央大學101學年度碩士班考試入學試題卷

所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁 第 1 頁
機械工程學系碩士班 乙組(製造與材料)(一般生)
機械工程學系碩士班 丙組(熱流)(一般生)
能源工程研究所碩士班 不分組(一般生)
機械工程學系光機電工程碩士班 乙組(光機)(一般生)

本科考試可使用計算器，廠牌、功能不拘

＊請在試卷答案卷（卡）內作答

Vector analysis and linear algebra

1. Given a scalar function $f(x,y)=x^2+y$,
   (a) Find the direction along which the directional derivative of f at (2, 1) is a maximum. (5%)
   (b) Find the directional derivatives of f at (2, 1) in the direction parallel to the tangent vector of the curve $x^2+2y^2=9$ at (1, 2). (5%)

2. Given a vector field $\boldsymbol{F}(x,y)=x^2\hat{i}+y\hat{j}$, find the direction along which $\left|\frac{dF(2,1)}{ds}\right|$ is a maximum, where s is the displacement. (8%)

3. Consider the linear system of equations $\mathbf{Ax}=\mathbf{b}$, where

$$\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 1 & m & 2\\ 2 & n & 1\end{bmatrix},\quad \mathbf{b}=\begin{bmatrix}2\\ 3\\ 3\end{bmatrix}.$$

Find the relation of m and n for which the problem has solution(s). (7%)

Partial differential equations and complex analysis

4. Find the deflection of the membrane of sides **a** and **b** with $c^2=1$ ($c^2=\frac{T}{\rho}$, where T is tension and ρ is the mass per unit length) from the two dimensional wave equation for the initial deflection $f(x,y)=xy(a-x)(b-y)$ and initial velocity 0. Deflections are zero on the boundary. (15%)

5. Flow with circulation around a cylinder can be represented by the complex potential : $F(z)=z+\frac{1}{z}-\frac{iK}{\pi}\ln z$. Calculate the stagnation point (points at which the velocity is zero) for K = 2π and 4π, respectively for which the cylinder wall $|z|=1$ is a streamline. (10%)

Ordinary differential equations

6. Consider the differential equation

$$x^2y''+(3x-1)y'+y=0.$$

(a) Show that $x=0$ is an irregular singular point. (5%)

(b) Show that

$$y=\sum_{n=0}^{\infty}n!\,x^n$$

is a series solution of the given differential equation. (5%)

7. A rabbit starts at the origin and runs up the y-axis with speed a. At the same time a dog, running with speed b, starting at the point $(c,0)$ and pursues the rabbit. Show that the path of the dog can be mathematically expressed by the differential equation,

$$xy'' = \frac{a}{b}\sqrt{1+(y')^2}. \qquad (10\%)$$

8. Let a sequence of functions $\phi_1(x)$, $\phi_2(x)$, ..., $\phi_n(x)$, ... be orthonormal functions on an interval $a \le x \le b$. Verify that

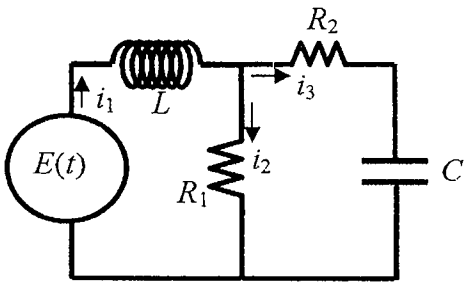
$$a_k = \int_a^b f(x)\phi_k(x)dx, \quad k = 1,2,...n,...$$

where $f(x)$ is an analytical function and a_k is the coefficient of expansions of the form representing $f(x)$ such that

$$f(x) = \sum_{k=1}^{\infty} a_k \phi_k(x) \qquad (5\%)$$

Laplace and Fourier transforms

9. An electric network is shown in Figure 1. L, C, R_1, R_2, and $E(t)$ represent inductor, capacitor, two resistors and external voltage, respectively. The currents in the network branches are i_1, i_2 and i_3 as indicated in the figure, respectively. The dynamical behavior can be expressed by the following differential equations:



$$L\frac{di_2}{dt} + L\frac{di_3}{dt} + R_1 i_2 = E(t)$$

$$-R_1\frac{di_2}{dt} + R_2\frac{di_3}{dt} + \frac{1}{C}i_3 = 0$$

Figure 1

(a) If the profile of $E(t)$ is shown in the Figure 2, Find the Laplace transform of $e^t E(t)$, $\mathcal{L}\{e^t E(t)\}$. (6%)

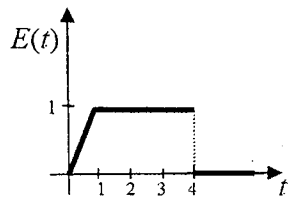


Figure 2

(b) If $E(t)$ can be formulated as $E(t) = e^{-kt}, k > 0$ and $t > 0$. Express $E(t)$ in terms of Fourier Cosine and Sine Integral representations respectively. (8%)

(c) Solve i_2 and i_3 if R_1=10 Ω, R_2=5 Ω, L=1 h, C=0.2 f, $E(t) = \begin{cases} 120 & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases}$, $i_2(0)$=0 and $i_3(0)$=0. (11%)

注：背面有試題