國立中央大學101學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生)　科目：基礎數學　共 1 頁 第 1 頁
統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

Q1: Linear Algebla (I) [30%] Let $A=\begin{bmatrix} 3 & -\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$.

(a) [5%] Find A^{-1}.

(b) [5%] Find all eigenvalues of A.

(c) [5%] Find the corresponding eigenvectors of A.

(d) [5%] Is A positive definite?

(e) [5%] Find matrices P and $\Lambda=\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ where $\lambda_1>\lambda_2$, $A=P\Lambda P^T$ and $PP^T=I_2$.

(f) [5%] Find $\min_{(x,y):x^2+y^2=1}\{3x^2-2\sqrt{2}xy+2y^2\}$ and its minimum point (x,y).

Q2: Linear Algebra (II) [10%] Let $L=\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$ where A and C have the inverse.

(a) [5%] Show that, if L is positive definite, then both A and C are positive definite.

(b) [5%] Let $L^{-1}=\begin{bmatrix} E & F \\ F^T & G \end{bmatrix}$. Write E and G in terms of A, B, and C.

Q3: Calculus (I) [10%] Let $0<p<1$.

(a) [5%] Find $\sum_{x=0}^{\infty}x(1-p)^{x-1}p$.

(b) [5%] Find $\sum_{x=0}^{\infty}(x-1/p)^2(1-p)^{x-1}p$.

Q4: Calculus (II)[30%] Let $f(x,y)=x^2+2y^2$.

(a) [5%] Find the maximum of $f(x,y)$ on the circle $x^2+y^2=4$.

(b) [5%] Find the minimum of $f(x,y)$ on the circle $x^2+y^2=4$.

(c) [5%] Show that $f(x,y)$ is a convex function.

(d) [5%]Find the maximum of $f(x,y)$ on the disc $x^2+y^2\le 4$.

(e) [5%]Find the minimum of $f(x,y)$ on the disc $x^2+y^2\le 4$.

(f) [5%]Find the maximum and minimum of $f(x,y)$ on $|x|+|y|\le 4$.

Q5: Calculus (III) [20%]

Evaluate $\int_{x=0}^{s}\int_{y=0}^{t}\{(1+\lambda x)(1+\lambda y)-\lambda\}\exp(-x-y-\lambda xy)dydx$.