科目：通訊系統(通訊原理)(300F)

校系所組：中央大學通訊工程學系（甲組）
中央大學電機工程學系（電子組）
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Fourier Transform: $X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt$

1. [14%] Assume $x(t)=\dfrac{\sin(100\pi t)}{\pi t}$ and $y(t)=3\cos(10^6\pi t+100\pi\int^{t}x(\alpha)d\alpha)$.

   (a) (4%) Find the bandwidth of x(t).

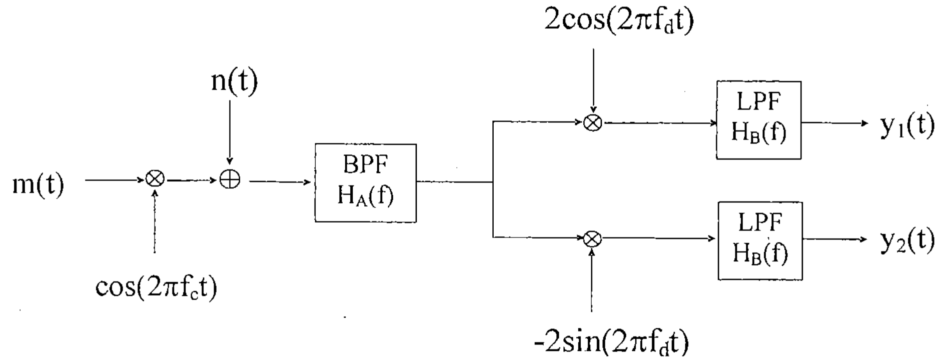
   (b) (3%) Find the total energy of x(t).

   (c) (4%) Based on Carson's rule, estimate the bandwidth of y(t).

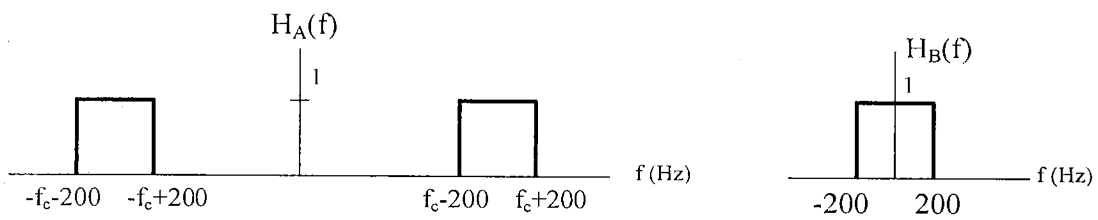
   (Carson's rule: Bandwidth ≈ 2(D+1)W, where W = bandwidth of x(t) and $D=\dfrac{\text{peak frequency deviation}}{W}$)

   (d) (3%) Find the power of y(t).

2. [20%] Assume a modulation/demodulation process is modeled as below where the input signal is $m(t)=(\dfrac{\sin(200\pi t)}{\pi t})^2$ while the additive channel noise n(t) is modeled as white Gaussian noise with two-sided power spectral density of 1×10^{-10} W/Hz.



In this process, the filters $H_A(f)$ and $H_B(f)$ are defined as



Assume the output signal $y_1(t)$ can be decomposed as $y_1(t) = y_{1s}(t) + y_{1n}(t)$, where $y_{1s}(t)$ and $y_{1n}(t)$ represent the signal part and the noise part of $y_1(t)$, respectively. Similarly, we have $y_2(t) = y_{2s}(t) + y_{2n}(t)$, where $y_{2s}(t)$ and $y_{2n}(t)$ represent the signal part and the noise part of $y_2(t)$, respectively.

Case 1: $f_c = f_d$

(a) (4%) For the signal part, find the spectrum of $y_{1s}(t)$ and $y_{2s}(t)$.

(b) (4%) For the noise part, find the power spectral density and the total power of $y_{1n}(t)$ and $y_{2n}(t)$.

(c) (2%) For the noise part, find the cross-power spectral density of $y_{1n}(t)$ and $y_{2n}(t)$.

Case 2: $f_c = f_d + 100$

(d) (4%) For the signal part, find the spectrum of $y_{1s}(t)$ and $y_{2s}(t)$.

(e) (4%) For the noise part, find the power spectral density and the total power of $y_{1n}(t)$ and $y_{2n}(t)$.

(f) (2%) For the noise part, find the cross-power spectral density of y1n(t) and y2n(t).

注意：背面有試題

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3. [13%] Let $X(t)$ be a wide-sense stationary random process with zero-mean and autocorrelation function $R_X(\tau) = E[X(t+\tau)X(t)] = e^{-2|\tau|}$. Suppose we feed $X(t)$ into a whitening filter with frequency response $H(f)$ and produce an output random process $Y(t)$ with autocorrelation function $R_Y(\tau) = \frac{1}{2}\delta(\tau)$, with $\delta(\tau)$ being the impulse delta function.
   (a) (4%) Please find the power spectral density and the average power of $X(t)$.
   (b) (4%) Please find the mean and variance of $X(t)|_{t=10}$.
   (c) (5%) Please find the magnitude response $|H(f)|$ of the whitening filter.

4. [16%] A binary communication system uses two waveforms $g_1(t)$ and $g_2(t)$. Both waveforms are time-limited to [0,T). Consider two cases where they are antipodal (i.e., $g_2(t) = -g_1(t)$) or orthogonal (i.e., $\int_0^T g_1(t)g_2(t)dt = 0$). Assume both waveforms have the same energy, i.e., $\int_0^T g_1^2(t)dt = \int_0^T g_2^2(t)dt$, and the noise is AWGN (additive white Gaussian noise).
   (a) (5%) Please design a correlation-based receiver for the antipodal case.
   (b) (5%) Please design a correlation-based receiver for the orthogonal case.
   (c) (6%) Which one (antipodal or orthogonal) has a lower bit error rate? Why?

5. [17%] Consider a QPSK transmission scenario. The transmitted constellation points (labeled by a, b, c, d) are shown in Fig. P5-1. The received constellation points are shown in Fig. P5-2 when the noise power is zero.
   (a) (6%) Please assign data bits to the QPSK constellation points via Gray encoding.
   (b) (6%) Please calculate the average transmitted symbol energy and the average received symbol energy.
   (c) (5%) Based on Figure P5-1 and Figure P5-2, please describe the channel effect of this QPSK transmission scenario.

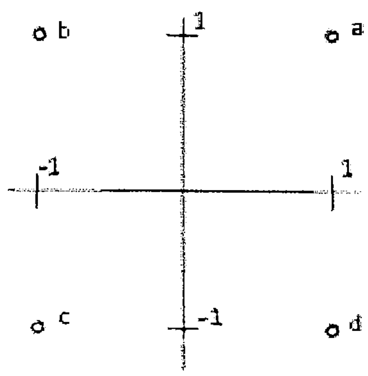

Figure P5-1

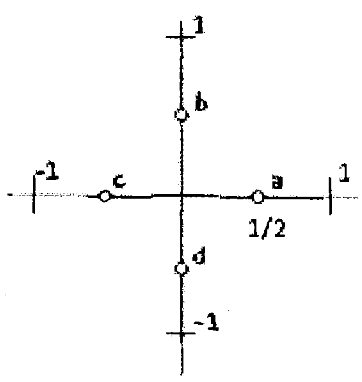

Figure P5-2

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參考用

6. [20%] Consider a two-user code-division multiple access system with BPSK modulated signal transmitted over AWGN (additive white Gaussian noise) channel. The $N \times 1$ received signal vector $\mathbf{y}$ observed at the basestation can be represented by

$$\mathbf{y} = A_1 b_1 \mathbf{s}_1 + A_2 b_2 \mathbf{s}_2 + \mathbf{w}$$

where, for $i=1,2$, $A_i > 0$ is the ith user's signal amplitude, b_i is the transmitted symbol equally likely to be -1 or $+1$, and the N-dimensional vector $\mathbf{s}_i$ is the ith user's signature sequence with the correlation property

$$\langle \mathbf{s}_i, \mathbf{s}_j \rangle \equiv \mathbf{s}_i^T \mathbf{s}_j = \begin{cases} 1, & \text{if } i = j, \\ \rho, & \text{if } i \neq j. \end{cases}$$

The noise vector $\mathbf{w}$ has i.i.d. Gaussian components with zero mean and variance σ^2. Assume that b_1, b_2 and $\mathbf{w}$ are statistically independent.

We consider the task of detecting user 1's symbol b_1 at the basestation receiver using a linear filter $\mathbf{c}$ that produces the statistic $\langle \mathbf{c}, \mathbf{y} \rangle = \mathbf{c}^T \mathbf{y}$ for decision making. Specifically, the receiver decides $\hat{b}_1 = +1$ if $\langle \mathbf{c}, \mathbf{y} \rangle \geq 0$ and decides $\hat{b}_1 = -1$ if $\langle \mathbf{c}, \mathbf{y} \rangle < 0$.

(a) (6%) Suppose the basestation aims to detect user 1's symbol b_1 using the matched filter $\mathbf{c}_{MF} = \mathbf{s}_1$ that produces the decision statistic $\langle \mathbf{c}_{MF}, \mathbf{y} \rangle = \mathbf{c}_{MF}^T \mathbf{y}$. Show that the bit error probability of the matched filter for user 1 is given by

$$P_e^{\mathrm{MF}} = \frac{1}{2} Q\left(\frac{A_1 - \rho A_2}{\sigma}\right) + \frac{1}{2} Q\left(\frac{A_1 + \rho A_2}{\sigma}\right),$$

where the Gaussian Q-function is defined as $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$.

(b) (4%) Following part (a), show that when $\rho > 0$ and $A_1 / A_2 < \rho$, the bit error probability has the asymptotic value $\lim_{\sigma \to 0} P_e^{\mathrm{MF}} = 1/2$. Please interpret the result.

(c) (4%) Suppose now the basestation employs the zero-forcing (ZF) filter $\mathbf{c}_{ZF}$ that satisfies $\mathbf{c}_{ZF}^T \mathbf{s}_1 = 1$ and $\mathbf{c}_{ZF}^T \mathbf{s}_2 = 0$ with the filter gain $\mathbf{c}_{ZF}$ taking the form $\mathbf{c}_{ZF} = \alpha_1 \mathbf{s}_1 + \alpha_2 \mathbf{s}_2$. Find the coefficients α_1 and α_2. Express your answer in terms of ρ.

(d) (6%) Following part (c), find the bit error probability P_e^{ZF} of the ZF receiver for user 1 and find the asymptotic value $\lim_{\sigma \to 0} P_e^{\mathrm{ZF}}$. Compare your result with that in part (b) and have some discussions.