科目：工程數學 D(3006)

校系所組：中央大學電機工程學系(電子組)
交通大學電子研究所(甲組)
交通大學電控工程研究所(甲組、乙組)

1. (16%)
   a) (8%) For an $n \times n$ upper triangular matrix A, prove that det(A) equals the product of the diagonal elements of A.

   b) (8%) Prove that if V is a vector space of dimension n, then any set of n linearly independent vectors spans V.

2. (10%) For the following matrix, find a basis for the row space and nullspace.

$$\begin{bmatrix} -1 & 2 & -3 \\ -1 & 4 & 7 \\ 2 & -5 & 1 \end{bmatrix}$$

3 (24%)
   a) (8%) For the system $A\underline{x} = \underline{b}$, find the least squares solution, where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -2 \\ -1 & 1 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 5 & -15 & -20 \end{bmatrix}$$

   b) (8%) Find the eigenvalues and corresponding eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

   c) (8%) Give the definition of similar matrix, and show that similar matrices have the same eigenvalues.

注意：背面有試題

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參考用

4. (5%) If $F(s)$ is the Laplace transform of $f(t)$, denoted by $F(s)=L\{f(t)\}$, find the inverse Laplace transform $L^{-1}\{F(as+b)\}$ in terms of $f(t)$, where $a>0$ and $b\neq 0$.

5. (5%) Solve $y'(t)=y(t)+4\int_0^t e^{-2(t-\tau)}y(\tau)d\tau,\ \ y(0)=1$.

6. (5%) Let $A=\begin{bmatrix}2 & -5\\ 1 & -2\end{bmatrix}$. Compute e^{At}.

7. (5%) Consider the non-homogeneous linear system $\underline{x}'=A\underline{x}+e^{\alpha t}\underline{v}$, where $\underline{x}$ is a vector consisting of functions in t, α is not an eigenvalue of A, and $\underline{v}\neq\underline{0}$ is a constant vector. Find a particular solution of the system, in terms of $A, \alpha, \underline{v}$ and t.

8. (10%)
   a) (5%) Determine the Fourier series coefficients (a_n, b_n) of the function $f(t)=t\cdot u(t)$ expanded over the interval $(-\pi, 2\pi)$, where $u(t)$ is the unit-step function.
   b) (5%) If the coefficients (a_n, b_n) from (a) are also the Fourier series coefficients of some function expanded over the interval $(-2\pi, 4\pi)$, find the function in terms of $f(t)$.

9. (12%) Given $y_1(x)=x^r$ is one solution of the homogeneous 2nd-order linear differential equation $x^2y''-5xy'+9y=0$.
   a) (2%) Derive its characteristics equation in terms of parameter r.
   b) (3%) Let $y_2(x)=v(x)y_1(x)$ be another linearly independent solution. Determine the governing differential equation of $v(x)$.
   c) (3%) Find $v(x)$ by solving the differential equation in (b).
   d) (4%) Apply the method of variation of parameters to find a particular solution of $y''-\frac{5}{x}y'+\frac{9}{x^2}y=x^2$.

10. (8%) Solve the differential equation $(x^2-1)y''-6xy'+12y=0$ by power series of the form $y=\sum_{n=0}^{\infty}c_nx^n$.
   a) (2%) Find the recurrence relation of c_n.
   b) (4%) Find the two linearly independent solutions. Please write the first three nonzero terms if it is an infinite series.
   c) (2%) State the guaranteed radius of convergence.