

甲、計算、證明題：共 3 大題，每大題 12 分，共 36 分。請在答案卷上詳細寫出計算及證明過程，否則不予計分。

1. Let $f(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Is it possible to define $f(0, 0)$ in a way that makes f continuous at the origin? Explain your answer and show your reasons. (Hint: Let $x = r \cos \theta$, $y = r \sin \theta$)

2. Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.

$$\oint_C x dx + y dy,$$

where C consists of the line segments from $(0, 1)$ to $(0, 0)$ and from $(0, 0)$ to $(1, 0)$ and the parabola $y = 1 - x^2$ from $(1, 0)$ to $(0, 1)$.

3. Determine if the given series converges or diverges. Explain your answer and show your reasons.

(a) $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n}).$

(b) $\sum_{n=2}^{\infty} \frac{2}{n \ln n}.$

乙、填充題：共 8 題，每題 8 分，共 64 分。請僅就答案部分依題號順序寫在答案卷上，不須呈現演算過程。

1. If $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \rightarrow -2} \left(f(x) + \frac{f(x)}{x} \right)$. Answer : _____

2. Find $\lim_{x \rightarrow 0} \frac{1}{3x^2} \int_{x^2}^0 \cos t dt$. Answer : _____

3. Find the rate of change of pressure at the point $P = (1, 2, 0)$ in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, where the pressure is given by $g(x, y, z) = xe^{-yz}$. Answer : _____

4. If $y = x^{\sin x}$, find the derivative of y with respect to x . Answer : _____

5. Evaluate $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4}{1+x^2+y^2} dx dy$. Answer : _____

6. Evaluate the double integral $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$. Answer : _____

7. Evaluate the definite integral $\int_{-1}^1 \frac{e^{\tan^{-1} y}}{1+y^2} dy$. Answer : _____

8. Find the work done by the force field $\mathbf{F}(x, y) = e^{-y}\mathbf{i} - xe^{-y}\mathbf{j}$ in moving an object from $P(0, 1)$ to $Q(2, 0)$. Answer : _____

參考用