科目 應用數學 類組別 035

＊請在試卷答案卷（卡）內作答

Please show the details of your work for all questions.

1. Test for exactness. If exact, solve. If not, find an integrating factor and then solve.

(a) $e^{-2\theta}\left(rdr - r^2 d\theta\right) = 0$ (10%)

(b) $\left(2\cos y + 4x^2\right)dx - x\sin y dy = 0$ (10%)

2. Solve the initial value problem.

$y'' + 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -5$ (10%)

3. The Gamma function is defined as $\Gamma(v) = \int_0^\infty e^{-t} t^{v-1} dt.$

Show that $\Gamma(v+1) = v\Gamma(v)$ and $\Gamma(1) = 1.$ (10%)

4. Derive the Laplace transform of $\cos\omega t$ and $\sin\omega t$. (10%)

[Hint : Consider the Laplace transform of $e^{i\omega t}$.]

注意：背面有試題

5. Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}. \quad (10\%)$$

6. Find curl $\vec{v}$, where $\vec{v} = \left[\ln\left(x^2 + y^2\right),\ 2\tan^{-1}\left(\frac{y}{x}\right),\ 0\right]$ is given with respect to right-handed Cartesian coordinates. (10%)

7. Find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, and $f(x) = x^2 \quad (-\pi < x < \pi)$. (10%)

8. Find the temperature $u(x, t)$ in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature $u(x, 0) = f(x)$. (10%)

9. $\int_{-L}^{L} \cos\frac{m\pi x}{L} \cos\frac{n\pi x}{L}\, dx = ?$ [m, n are integer.] (10%)