

甲、選擇題：共 8 題，每題 6 分，共 48 分。請用大寫字母 A, B, C, D 或 E 答題，並將答案依題號順序寫在答案卷上。皆單選。

1. Use the fact that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ to find the value of $\lim_{x \rightarrow 0} \frac{2x}{\tan 7x}$.

(A) ∞ (B) 0 (C) 1 (D) $\frac{2}{7}$ (E) $\frac{7}{2}$

2. For what values of a, m , and b does the function $f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

- (A) $a = 3, m = 1, b = 4$ (B) $a = 3, m = 2, b = 3$ (C) $a = 3, m = 4, b = 1$
 (D) $a = 2, m = 1, b = 3$ (E) None of the above

3. Define $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$. Find $f'(0)$

- (A) Does not exist (B) 1 (C) 2 (D) $\frac{1}{2}$ (E) 0

4. Assume $x = 2 \tan t, y = \sec^2 t - 1$. Find an equation for the line tangent to the curve at the point where $t = \frac{\pi}{4}$.

- (A) $y = -x - 1$ (B) $y = -x + 1$ (C) $y = x$ (D) $y = x + 1$ (E) $y = x - 1$

5. Find the limit: $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t \, dt$

(A) ∞ (B) $-\infty$ (C) -1 (D) 0 (E) 1

6. Find the limit: $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$.

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2 (E) Does not exist

7. Find the line integral of $F = (2xyz, x^2z, x^2y)$ over any path from $(0, 0, 0)$ to $(1, 2, 3)$?

(A) 6 (B) 9 (C) 12 (D) 18 (E) None of the above

8. Find the surface area of the portion S of the cone $z^2 = x^2 + y^2$, where $z \geq 0$, contained within the cylinder $y^2 + z^2 \leq 1$.

- (A) 0 (B) π (C) $-\pi$ (D) 2π (E) -2π

乙、填充題：共 4 題，每題 8 分，共 32 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Evaluate the indefinite integral $\int \frac{1}{1+e^x} dx$.

Answer : _____

2. Find the points on the graph of $z = 3x^2 - 4y^2$ at which the vector $\mathbf{n} = \langle 3, 2, 2 \rangle$ is normal to the tangent plane.

Answer : _____

3. What is the value of the double integral $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1} dx dy$?

Answer : _____

4. What is the largest value that the directional derivative of $f(x, y, z) = xyz$ can have at the point $(1, 1, 1)$?

Answer : _____

丙、計算、證明題：共 2 大題，每大題 10 分，共 20 分。須詳細寫出推論與演算過程，否則不予計分。

1. (a) Determine if the series $\sum_{n=1}^{\infty} (-1)^n \cos \frac{1}{n}$ converges or diverges? (5 分)

(b) Determine if the improper integral $\int_0^1 \frac{dx}{x - \sin x}$ converges or diverges? (5 分)

2. Find the maximum area of a rectangle inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$. (10 分)