

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. If $f'(0) = -1$, find $\lim_{h \rightarrow 0} \frac{f(3h) - f(-2h)}{h}$. Answer : _____
2. Find $\frac{d^2}{dx^2} \int_0^x \left(\int_1^{\sin t} \sqrt{1+u^4} du \right) dt$. Answer : _____
3. Convert the integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 dz dy dx$ to an equivalent integral in cylindrical coordinates. Answer : _____ (Do not evaluate the integral).
4. Evaluate the integral $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$. (Hint: $\frac{e^{-ax} - e^{-bx}}{x} = \int_a^b e^{-xy} dy$)
Answer : _____
5. Find the point on the graph of $z = x^2 + y^2 + 10$ nearest the plane $x + 2y - z = 0$.
Answer : _____
6. The derivative of $f(x, y)$ at $P_0(1, 2)$ in the direction of $\mathbf{i} + \mathbf{j}$ is $2\sqrt{2}$ and in the direction of $-2\mathbf{j}$ is -3 . What is the derivative of f in the direction of $-\mathbf{i} - 2\mathbf{j}$? Answer : _____
7. Find the area of cap cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cone $z = \sqrt{x^2 + y^2}$.
Answer : _____
8. Evaluate the line integral $\oint_C \frac{y}{x^2 + y^2} dx - \frac{x}{x^2 + y^2} dy$ where C consists of line segments joining successively the points $(1, 0)$, $(1, 1)$, $(-1, 1)$ and $(-1, 0)$. Answer : _____

乙、計算、證明題：共 3 大題，每大題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$
 - (a) Show that $f'(0) = 0$. (6 分)
 - (b) Is the derivative function $f'(x)$ continuous at $x = 0$? Explain. (6 分)
2. (a) Find the limit: $\lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} \sin \sqrt{t} - \sqrt{t} dt}{\int_0^{x^2} \tan \sqrt{t} - \sqrt{t} dt}$. (6 分)
- (b) Let a and b be constants with $0 < a < b$. Does the sequence $\{(a^n + b^n)^{1/n}\}$ converges? If it does converge, what is the limit? (6 分)
3. (a) A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x , then f has at most one fixed point. (6 分)
- (b) Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \left(\frac{1 + \sin(a_n)}{2} \right)^n$ converges. (6 分)